p-adic mathematical physics and Branko Dragovich works

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Topics

- p-Adic and adelic strings
- Summation of p-adic series in rational points
- p-Adic and adelic quantum mechanics
- p-Adic and adelic gravity and quantum cosmology
- p-Adic genetic code
p-Adic and adelic strings


\[ \mathcal{A}(a, b) = \int dx \gamma_a(x) \gamma_b(1-x) \]

\[ \mathcal{A}_p(a, b, c) = \frac{\Gamma_p(a) \Gamma_p(b)}{\Gamma_p(a+b)} \]

\[ \Gamma_p(y) = \frac{1-p^{y-1}}{1-p^{-y}} \]

I.V., Freund, Witten, Frampton, … Gelfand-Graev-Piatetski-Shapiro
Veneziano string amplitude

\[ B(a, b) = \int_{-\infty}^{+\infty} |x|^{a-1} |1-x|^{b-1} \, dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} + \frac{\Gamma(b)\Gamma(c)}{\Gamma(c+b)} + \frac{\Gamma(c)\Gamma(a)}{\Gamma(c+a)} \]

\[ a + b + c = 1 \]

Mandelstam variables

\[ B(a, b) = \frac{\zeta(1-a)}{\zeta(a)} \frac{\zeta(1-b)}{\zeta(b)} \frac{\zeta(1-c)}{\zeta(c)} \]

Aref'eva, Dragovich, I.V., 1988
p-adic string amplitudes

\[ B_p(a, b) = \int_{Q_p} |x|^a \cdot |1 - x^b| \, dx = \frac{1 - p^{a-1}}{1 - p^{-a}} \cdot \frac{1 - p^{b-1}}{1 - p^{-b}} \cdot \frac{1 - p^{c-1}}{1 - p^{-c}} \]

\[ \prod_{p=\infty, 2, 3, \ldots} |x|^p = 1, \quad \text{Adelic product} \]

\[ \prod_p |B_p(a, b)| B_\infty(a, b) = \frac{\zeta(-a) \zeta(-b) \zeta(a+b)}{\zeta(a) \zeta(b) \zeta(-a-b)} \]

\[ \prod_p B_p(a, b) B_\infty(a, b) = 1 \quad \text{Freund-Witten (nice but divergent)} \]

Aref’eva, Dragovich, I.V.
p-adic superstrings
(Aref'eva-Dragovich-I.V.-Vladimirov)

P-adic conformal theory

\[ \langle x^\mu(z)x^\nu(w) \rangle = -g^{\mu\nu} \log |z-w|^2_k \]

P-adic 4-fermion string amplitudes

\[
A_{4F}^{(s)}(k_1 u_1; k_2 u_2; k_3 u_3; k_4 u_4) = -\frac{1}{2} g^2 u_1^\alpha u_2^\beta u_3^\gamma u_4^\delta \int \frac{dz}{Q_p} \theta_{\tau,[0,1]}(z) \left| z \right|^{1-t/2} \left| 1-z \right|^{1-s/2} \left| 1-z \right|_p \gamma^\mu_{\alpha\beta} \gamma_{\gamma\delta}^\mu - \left| z \right|_p \gamma^\mu_{\alpha\delta} \gamma_{\beta\gamma}^\mu \right]
\]

\[
= u_1^\alpha u_2^\beta u_3^\gamma u_4^\delta ( -\frac{1}{2} g^2 ) (1-1/p) \frac{1+p^{-u/2-1}}{1+p^{u/2}}
\]

\[
x \left( \frac{p^{t/2}-p^{-t/2-1}+p^{s/2-1}-p^{-s/2}}{(1-p^{t-2})(1-p^{-t})} \gamma_{\alpha\beta}^\mu \gamma_{\gamma\delta}^\mu + \frac{p^{s/2}-p^{-s/2-1}+p^{t/2-1}-p^{-t/2}}{(1-p^{t-2})(1-p^{-s})} \gamma_{\alpha\delta}^\mu \gamma_{\beta\gamma}^\mu \right)
\]
\[
\theta_{\epsilon, [0, 1]}(z) = \frac{1}{2} \left[ \text{sign}_\epsilon z - \text{sign}_\epsilon (-1) \text{sign}_\epsilon (1 - z) \right]
\]

\[
\text{sign}_\epsilon z \quad \text{Multiplicative character of Qp related with} \quad Q_p(\sqrt{\epsilon})
\]

Gelfand – Graev – Piatecki-Shapiro

V.S. Vladimirov, for 5 types of superstrings
Kozyrev, Vladimirov, Khrennikov, I.V., Dragovich, Kochubei

B.Dragovich: Nonlocal zeta scalars and string models.

A New Model of Nonlocal Modified Gravity
Ivan Dimitrijevic, Branko Dragovich, Jelena Grujic, Zoran Rakic
Summation of $p$-adic series in rational points


Power series everywhere convergent on $\mathbb{R}$ and all $\mathbb{Q}_p$

Branko G. Dragovich
Adelic quantum mechanics

\[(L_2(\mathbb{R}), \, W(z_{\infty}), \, U(t_{\infty}))\]

\[(L_2(\mathbb{Q}_p), \, W(z_p), \, U(t_p))\] p-adic quantum mechanics, Vladimirov, Zelenov, I.V.

Adelic: Manin, Dragovich,…

\[(L_2(\mathbb{A}), \, W(z), \, U(t))\]
p-adic quantum mechanics

“p-Adic and Adelic Harmonic Oscillator with Time-Dependent Frequency”

Goran S. Djordjevic, Branko Dragovich
Lecture 10: p-Adic and Adelic Dynamical Systems

- The state space $X$ may have some additional structures, e.g. hierarchies and ultrametric distances, which can be described by p-adic numbers.
- The same ultrametric states may have different p-adic properties and then their description needs adeles.
- We consider here an adelic state space $X = \mathbb{A}$ with discrete evolution described by linear fractional function

$$f(x) = \frac{ax + b}{cx + d}, \quad x, f(x) \in \mathbb{A}, \quad a, b, c, d \in \mathbb{Q}, \quad ad - bc \neq 0,$$

and in particular $ad - bc = 1$. 

Giving series of lectures on p-Adic Mathematical Physics and its Applications, PhD students at School of Mathematics, Shandong University, Jinan, China, June 2012
Journal “p-Adic Nimbers, Ultrametric analysis and Applications”

Branko Dragovich is Deputy Editor-in-Chief
DEAR BRANKO, DEAR ORGANIZERS
THANK YOU - HERZLICHEN DANK
HVALA / ХВАЛА - БЛАГОДАРЯ

Endurance is one of the most difficult disciplines, but it is to the one who endures that the final victory comes.

-Gautama Buddha

A great man is he who endures, and not he who is strong, for strength itself is insufficient.

-Wolfgang von Goethe

You can only lead if you want to lead. You can only control if you want to control. You have to lead in order to lead. You have to control in order to control.

-Flavius Josephus

...and to give than to receive.

-Acts (ch. XX, v. 25)
The wave function of the universe and p-adic gravity
Arefeva, Dragovich, Frampton, I.V. (Int. J. Mod. Phys, 1991)

Penrose - Hawking

\[ \Psi(G^3) = \sum_{\text{manifolds}} \int e^{-S(g)} Dg_{\mu\nu} \]

ADFV

\[ \Psi(G^3) = \sum_{\text{algebraic manifolds}} \int \prod_p \chi_p(S_p) D(g_{\mu\nu})_p \]
ACS discovers two distant Type Ia supernovae
Purpose of human existence

• According to the anthropic principle there are many universes and our universe is just one of them better suitable to support life as we know it.

• The purpose of human existence is to save our Universe. In particular to prevent a possible collapse, Big Crunch or Big Freeze (Branko Dragovich)