Renormalizable ghost-free gravity

Alexey Koshelev

Universidade da Beira Interior & Vrije Universiteit Brussel

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Happy Birthday, Branko!

The set-up

We start with GR, which must be the IR limit anyway

$$S = \int d^4x \sqrt{-g}\left(\frac{M_P^2 R}{2}\right), \;\; M_P^2 = \frac{1}{8\pi G_N}$$

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WHY???

History

- **Classical gravity and GR; also Ostrogradski 1850**
- **Stelle, 1977,1978, renormalizable R^2 type gravity (containing ghosts)**
- **Starobinsky, 1980-s, R^2 inflation**
- **Witten, 1986, String Field Theory (SFT) which by construction contains non-local vertexes**
- **Vladimirov, Volovich, Zelenov; Dragovich, Khrennikov; Brekke, Freund, Olson, Witten, . . . , 1987+, p-adic strings, again non-local**
- **Aref'eva, AK, 2004, models of non-local stringy inspired scalar fields coupled to gravity**
- **Biswas, Mazumdar, Siegel, 2005, first explicit non-local gravity modification**
- **Recent activity by Biswas, Conroy, Dragovich, Koivisto, Mazumdar, Modesto, Pozdeeva, Rachwal, Vernov, AK and others**

Problems of GR

- **GR does not explain the Dark Energy**
  Or why the sky is dark in the night? Zel'dovich

- **GR is not renormalizable**
  Obvious due to the dimensionful coupling M_P^2

- **GR is not geodesically complete**

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- **GR is not geodesically complete**

- **Either of the above has been overcome sacrificing the unitarity**

  Ghosts appeared in the theory

Why Einstein's GR is not enough?

- **Raychaudhuri equation:**
  Consider a congruence of null geodesics characterized by a null vector k_α, such that $k_\alpha k^\alpha = 0$. Then

$$R_{\mu\nu}k^\mu k^\nu < 0$$

for a non-singular space-time

- **GR equations of motion are:**

$$M_P^2 G_{\mu\nu} = T_{\mu\nu}$$

Assuming the matter is a perfect fluid

$$T_\nu^\mu = \mathrm{diag}(-\rho, p, p, p) \;\Rightarrow\; T_{\mu\nu}k^\mu k^\nu = \rho + p$$

- **Then**

$$M_P^2 R_{\mu\nu}k^\mu k^\nu = \rho + p$$

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- **Then**

$$0 > M_P^2 R_{\mu\nu}k^\mu k^\nu = \rho + p > 0$$

Either the space-time is singular or the NEC is violated.

Who are ghosts?

Terminology in $(-,+,+,+)$ signature:

$$\textbf{good:}\ L = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \dots$$
$$\textbf{ghost:}\ L = +\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \dots$$

Ghosts lead to a very rapid vacuum decay.

Ostrogradski statement says that higher (> 2) derivatives in a Lagrangian are equivalent to the presence of ghosts.

This statement is not absolutely rigorous. There are systems with higher derivatives which have no ghosts.

Exorcising ghosts

- **In some cases ghosts do not appear, like in $f(R)$ gravity for special parameters.**

  This is because the system is constrained.

- **There are special field theories which have higher derivatives in the Lagrangian but no more than 2 derivatives act on a field in the equations of motion. For example KGB models or galileons.**

  The fine-tuning is required.

- **Propagators can be modified and be non-local without changing the physical excitations**

$$\Box - m^2 \rightarrow \mathcal{G}(\Box) = (\Box - m^2)e^{\gamma(\Box)}$$

  $\gamma(\Box)$ **must be an entire function. This guarantees that no extra degrees of freedom appear. Let** $\gamma(0) = 0$ **to preserve the normalization.**

$\mathcal{G}(\Box)$ physics, SFT motivation

Low level example action from SFT:

$$L \sim \frac{1}{2}\phi(\Box - m^2)\phi + \frac{\lambda}{4}\left(e^{-\beta\Box}\phi\right)^4 \Rightarrow \frac{1}{2}\varphi(\Box - m^2)e^{2\beta\Box}\varphi + \frac{\lambda}{4}\varphi^4$$

The Lagrangian to understand is

$$S = \int d^D x \left(\frac{1}{2}\varphi\mathcal{G}(\Box)\varphi - \lambda v(\varphi) + \dots\right)$$

$\mathcal{G}(\Box) = \sum_{n\geq 0} g_n \Box^n$, **i.e. it is an analytic function.**

Canonical physics has $\mathcal{G}(\Box) = \Box - m^2$, **i.e.** $L = \frac{1}{2}\varphi\Box\varphi - \frac{m^2}{2}\varphi^2$

Ghosty example $\mathcal{G}(\Box) = \Box - m^2 + g_2\Box^2$

Coming back to the advertised setting

$$S=\int d^4x\sqrt{-g}\left(\frac{M_P^2R}{2}+\frac{\lambda}{2}\left(R\mathcal{F}_1(\Box)R+R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu}+R_{\mu\nu\lambda\sigma}\mathcal{F}_4(\Box)R^{\mu\nu\lambda\sigma}\right)\right)$$

Linearization around Minkowski space-time, $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu},\ h=h^\mu_\mu$**:**

$$\delta^{(2)}S=\int\frac{d^4x}{2}\left(\frac{1}{2}h_{\mu\nu}\Box\alpha(\Box)h^{\mu\nu}+\partial_\sigma h^\sigma_\mu\alpha(\Box)\partial_\nu h^{\mu\nu}+h\gamma(\Box)\partial_\mu\partial_\nu h^{\mu\nu}-\frac{1}{2}h\Box\gamma(\Box)h\right.$$
$$\left.-\partial_\alpha\partial_\beta h^{\alpha\beta}\frac{\gamma(\Box)-\alpha(\Box)}{\Box}\partial_\mu\partial_\nu h^{\mu\nu}\right)$$

$$\alpha(\Box)=M_P^2-\frac{\lambda}{2}\Box\mathcal{F}_2(\Box)-2\lambda\Box\mathcal{F}_4(\Box),\ \ \gamma(\Box)=M_P^2+2\lambda\Box\mathcal{F}_1(\Box)+\frac{\lambda}{2}\lambda\Box\mathcal{F}_4(\Box)$$

$$\gamma(\Box)-\alpha(\Box)=2\lambda\Box[\mathcal{F}_1(\Box)+\frac{1}{2}\mathcal{F}_2(\Box)+\mathcal{F}_4(\Box)]$$

Notice: a generalized Gauss-Bonnet term $\mathcal{F}_1(\Box)=-4\mathcal{F}_2(\Box)=\mathcal{F}_4(\Box)$ **always gives** $\gamma(\Box)=\alpha(\Box)$

Propagator

Projection operators (van Nieuwenhuizen, 1973):

$$\mathcal{P}^2 = \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\nu\rho}\theta_{\mu\sigma}) - \frac{1}{3}\theta_{\mu\nu}\theta_{\sigma\rho}, \ \mathcal{P}^0_s = \frac{1}{3}\theta_{\mu\nu}\theta_{\sigma\rho}, \ \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

Plus two more which are not relevant here.

The propagator:

$$\Pi = \frac{\mathcal{P}^2}{\alpha(-k^2)k^2} + \frac{\mathcal{P}^0_s}{(\alpha(-k^2) - 3\gamma(-k^2))k^2}$$

Recall the pure GR propagator:

$$\Pi_{GR} = \frac{\mathcal{P}^2}{k^2} - \frac{\mathcal{P}^0_s}{2k^2}$$

1. Absence of new degrees of freedom requires $\alpha(-k^2)$ **and** $\alpha(-k^2) - 3\gamma(-k^2)$ **have no roots**

2. Presence of a GR limit requires $\alpha(0) = \gamma(0) = 1$

Example

- **Non-local terms can be chosen as:**

$$\mathcal{F}_4(\Box) = 0, \ \mathcal{F}_1(\Box) = -\frac{1}{2}\mathcal{F}_2(\Box)$$

$$\mathcal{F}_1(\Box) = \frac{e^{\sigma(\Box)} - 1}{\Box}, \ \sigma(\Box) \textbf{ is an entire function and } \sigma(0) = 0$$

- **This leads to a manifestly asymptotically-free gravity:**

$$\sigma(\Box) = -\frac{\Box}{M}, \ \Phi \sim -\frac{1}{r}\mathrm{erf}\left(\frac{Mr}{2}\right) \to \begin{cases} \text{const as } r \to 0 \\ \frac{1}{r} \text{ as } r \to \infty \end{cases}$$

- **It is also singularity-free following the Raychaudhuri equation analysis**
  Conroy, AK, Mazumdar, PRD, 2014 and a joint work in progress

Solutions of FRW type

First we reshuffle the non-local terms by using the Weyl tensor

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2}+\frac{\lambda}{2}\left(R\tilde{\mathcal{F}}_1(\Box)R + R_{\mu\nu}\tilde{\mathcal{F}}_2(\Box)R^{\mu\nu} + C_{\mu\nu\lambda\sigma}\tilde{\mathcal{F}}_4(\Box)C^{\mu\nu\lambda\sigma}\right)\right)$$

To satisfy the conditions obtained from the consideration of the propagator one can set

$$\tilde{\mathcal{F}}_2(\Box) = 0,\ \tilde{\mathcal{F}}_1(\Box) = -\frac{1}{3}\tilde{\mathcal{F}}_4(\Box)$$

Claim: any solution of the local R^2 gravity is a solution here upon 3 algebraic conditions on the action parameters

Accounting $\Box R = r_1 R + r_2$ (which is an EOM rather than a constraint in a local R^2 gravity) and letting the cosmological term Λ to be in the action

$$\mathcal{F}^{(1)}(r_1) = 0,\ \frac{r_2}{r_1} = -\frac{M_P^2 - 6\lambda\mathcal{F}(r_1)r_1}{2\lambda[\mathcal{F}(r_1) - \mathcal{F}(0)]},\ \Lambda = -\frac{r_2 M_P^2}{4r_1},$$

Examples of solutions

Let as usual a is the scale factor of the FRW metric

Explicit non-singular bouncing solutions

$$a = a_0 \cosh(\sigma t)$$

Biswas, Muzumdar, Siegel, JCAP, 2006; AK, CQG, 2013

$$a = a_0\sqrt{\cosh(\sigma t)} \textbf{ and also } a = a_0 e^{-\frac{\sigma}{2}t^2}$$

AK, CQG, 2013

Starobinsky solution

$$a \approx a_0\sqrt{t_* - t}e^{\sigma(t_*-t)^2}$$

Craps, De Jonckheere, AK, JCAP, 2014

Quantization of perturbations was studied and shown to reproduce the values close to observable in the cosmological experiments

A wishful solution

In contrast with the local R^2 gravity one can arrange such a parameter range that both bounce type and inflation type solutions coexist.

We are however lack of an explicit construction of such a solution yet.

AK, work in progress

Renormalizability

The renormalizability must follow since the newly emerged momentum-dependent factors yield an exponential suppression in the UV.

An example, for us a toy-model, we have in mind is the p-adic string theory proven to be manifestly finite. Its Lagrangian is

$$L = -\frac{1}{2}\phi p^{-\Box/m_p^2}\phi + \frac{1}{p+1}\phi^{p+1}$$

Conclusions

- **Non-local SFT motivated generalization of Einstein's gravity is presented**

- **The ghost-free conditions on the propagator are clearly formulated**

- **There are exist exact analytic solutions including bounce and the Starobinsky inflation in this framework**

Open questioins

- **Study of generalized models of the non-local gravity**
  I.Dimitrijevic, B.Dragovich, J.Grujic, AK, Z.Rakic, to appear next week

- **There are preliminary results for the graviton action in the bosonic closed SFT**
  AK, work in progress

- **The next major step is to derive in some lowest approximation an action for the massless states in a heterotic SFT**

Thank you for listening!