BRANKO DRAGOVICH

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Abstract

We present a brief biography and a brief review of the scientific work of
Branko Dragovich on the occasion of his 70th birthday on March 11, 2015.

1 Brief Scientific Biography

Branko Dragovich was born on March 11, 1945 in the village Sonković (near Šibenik,
Croatia, former Yugoslavia). Since 1964, he lives in Belgrade, Serbia. He completed
his primary school in Skradin (1959), graduated grammar school in Šibenik (1963).
He completed his studies in physics (1967), defended his MSc in theory of elementary
particles (1973) and PhD in quantum field theory (1977), all at the Faculty of Science
and Mathematics in Belgrade.

B. Dragovich had tenured position in the Institute of Physics, University of Belgrade, Belgrade, from 1970 until retirement in 2012. He was the first director of the
Center for Theoretical Physics (1978-1985) of the Institute of Physics, Belgrade. He
is research professor since 1994. He was also employed in the Steklov Mathematical
Institute of the Russian Academy of Sciences in Moscow, 2000-2002. He spent in
Steklov Mathematical Institute, Moscow, and Joint Institute for Nuclear Research,
Dubna (Russia), in total about 5 years. He has visited many other foreign institutions in physics and mathematics (in total more than 80 times), in particular:
International Centre for Theoretical Physics, Trieste, Italy; Department of Mathematics of the Linnaeus University, Växjö, Sweden; Institute for Applied Mathematics, Bonn University, Germany; Theory Division, CERN, Geneva, Switzerland;
Department of Physics and Astronomy, University of North Carolina, Chapell Hill,
USA; Department of Theoretical Physics, Australian National University, Canberra,
Australia; School of Mathematics, Shandong University, Jinan, China; Sogang University, Seoul, South Korea; Laboratory of Theoretical Physics, Annecy, France.

B. Dragovich was principal investigator on the international projects: 1) "Mathematical Methods of Elementary Particle Theory", sponsored by the the Institute of
Physics (Belgrade) and the Steklov Mathematical Institute (Moscow), 1981–1990,
and 2) "Modelling of the Genetic Code and Applications", granted by the Ministry
of Science and Technological Development, Serbia and CNRS, France, 2009–2010.
He was also head of the project "Quantum Models on Noncommutative and Adelic
Spaces", granted by Ministry of Science of the Republic of Serbia, 2002–2005. B.
Dragovich participated in Serbian scientific projects: "Fundamental and Methodological Problems of Physics", "Geometry, Education and Visualization with Applications" and "Bioinformatic Promoter Predictions and Theoretical Modeling of

Gene Circuits in Bacteria". He was also a member of the projects No 990100866 and No 02-01-01084 supported by the Russian Fund of Fundamental Research.

B. Dragovich is deputy editor-in-chief of the international journal "p-Adic Numbers, Ultrametric Analysis and Applications" and performs its technical work before publication. He was main organizer of 12 international scientific meetings, including series of summer schools and conferences on modern mathematical physics held in Serbia since 2001, and participated in organization of 35 other scientific conferences. B. Dragovich was editor or coeditor of 22 books, and a referee in more than 30 international journals. He participated in more than 130 international meetings, mainly giving talk by invitation.

B. Dragovich was twice awarded by research prize of the Institute of Physics, Belgrade, for (i) contribution to finite quantum electrodynamics (1983) and (ii) for results in adelic quantum models and summation of p-adic series (1996). He was also awarded by special Charter of the Institute of Physics (Belgrade) for contribution to its development (1981). In occasion of his 60th birthday, the *Proceedings of the 2nd International Conference on p-Adic Mathematical Physics* were dedicated to him, and published by AIP Conference Proceedings **826** (New York, 2006). B. Dragovich is a member of South-Slavic Academy of Nonlinear Sciences since November 2011 and its president since June 2014.

Parallel to research activity, B. Dragovich participated in teaching and taught courses in many fields of theoretical physics, in particular in: Elementary Particle Physics, Classical and Quantum Electrodynamics, Quantum Field Theory, Astrophysics, Gravity and Cosmology. The courses were at the University of Belgrade, the University of Priština, the University of Niš and the University of Banja Luka. He gave by invitation also some special courses in Sweden, China and South Korea. B. Dragovich was supervisor of 5 PhD theses (Branislav Sazdovic, Goran Djordjevic, Ljubisa Nesic, Jelena Grujic, Ivan Dimitrijevic), 5 Magister theses and a number of Diploma works, all in theoretical and mathematical physics. He gave more than 120 seminar talks in Serbian, English and Russian language.

2 Brief Review of Scientific Research

Since 1987, after I. Volovich publication on p-adic strings [1], B. Dragovich research has been mainly devoted to *p-Adic Mathematical Physics* (for a review in different times, see, e.g. [2, 3, 4, 5, 6] and references therein). We will first present a brief review of B. Dragovich research in p-adic mathematical physics and after that his contribution to some other research fields.

1. p-Adic and adelic strings. p-Adic mathematical physics practically started by construction of amplitudes for scattering of scalar p-adic strings [1]. Branko Dragovich, with Irina Aref'eva and Igor Volovich, constructed and investigated some new amplitudes for scattering of strings and superstrings [7, 8, 9]. B. Dragovich also constructed and investigated a few effective Lagrangians with the Riemann zeta function, which are related to the whole p-adic sector of open scalar strings [10, 11]. Note that p-adic strings play an important role in string theory and modern cosmology.

2. Summation of p-adic series in rational points. In theoretical physics we often face the problem of divergent perturbation series. When coefficients of such power series are rational numbers, then they can be treated as not only real but also as p-adic series. This motivated Aref'eva, Dragovich and Volovich to investigate convergence and summation in rational points divergent series of the anharmonic oscillator [12]. B. Dragovich continued with research in this direction and investigated a broad class of power series with factorials in their coefficients and found the corresponding sums in rational points [13, 14, 15, 16, 17].

3. p-Adic and adelic quantum mechanics. Adeles provide a comprehensive approach to theoretical modelling taking into account possible numbers which are completion of rationals and possible geometries, and offer to understand the role so far preferred by real numbers and Archimedean geometry. Using adelic analysis, B. Dragovich formulated adelic quantum mechanics [18, 19], which unifies both standard and all p-adic quantum mechanics. It is a new research direction of quantum phenomena, wider and more complete than standard quantum mechanics, which can be obtained as a special case of the adelic one. As an interesting result, it should be noted that B. Dragovich found link between adelic harmonic oscillator and the Riemann zeta function [18, 19]. He considered also some generalized adelic functions [20], which appear in adelic quantum mechanics [21].

Feynman path integral plays an important role in standard and p-adic quantum mechanics. B. Dragovich generalized Feynman path integral to the case of adelic quantum mechanics, extending integration over standard real paths to all possible p-adic path analogs. With his collaborators, Goran Djordjevic and Ljubisa Nesic, B. Dragovich obtained exact expression of Feynman path integral for quantum systems with quadratic Lagrangians [22, 23], which has the same form as in standard quantum mechanics. This means invariance of Feynman path integral under change of real and p-adic number fields, what is in agreement with *Volovich invariance principle* [24] that fundamental physical laws should be number field invariant. Adelic quantum mechanics has been applied to investigation of some simple systems, and in particular, to harmonic oscillator [18, 19], harmonic oscillator with time-dependent frequency [25] and a free relativistic particle [26].

4. p-Adic and adelic gravity and quantum cosmology. p-Adic gravity and adelic quantum cosmology are introduced in common paper with I. Aref'eva, P. Frampton and I. Volovich [27]. Later, B. Dragovich with collaborators developed adelic quantum cosmology as a part of adelic quantum mechanics, see [28, 29]. As a consequence of adelic approach and p-adic effects, adelic quantum cosmology contains discreteness of space-time at the Planck scale.

5. p-Adic genetic code. The genetic code is connection between 64 codons, which are building blocks of genes in DNA, and 20 amino acids, which are constituents of proteins. It is interesting that the codon space has ultrametric structure, which can be well described by p-adic distance. B. Dragovich used 5-adic and 2-adic distance to identify codons which code the same amino acid [30, 31]. p-Adic distance approach is very promising to study similarities in bioinformation systems.

6. Other researches. With I. Aref'eva and I. Volovich, B. Dragovich investigated supergravity equations with some extra time-like dimensions and obtained solutions with vanishing cosmological constant [32, 33]. B. Dragovich considered change of

signature in p-adic space-times [34], see also [35].

B. Dragovich investigated finite quantum electrodynamics and its ultraviolet divergences [36]. It was shown that the corresponding Dyson-Schwinger equation for electron propagator does not lead to its electromagnetic mass generation [37], but see [38, 39] when it is possible.

Path integral in noncommutative quantum mechanics for quadratic Lagrangians was investigated with Zoran Rakic [40]. Possible dynamical control of decoherence by perpendicular electromagnetic field on noncommutative plane was considered with Miroljub Dugić [41].

p-Adic and adelic linear fractional dynamical systems are studied with Andrei Khrennikov and Dušan Mihajlović [42]. B. Dragovich also considered p-adic and adelic superanalysis [43] and possibility to apply p-adic analysis in econometrics [44].

Recently, B. Dragovich participates in investigation of nonlocal modified gravity and its cosmological solutions, see, e.g. [45]. He also works towards p-adic origin of dark matter and dark energy [46].

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At the end, we would like to emphasize that Branko Dragovich is deeply interested in fundamental problems and has made important contributions not only in physics but also in life science. He is one of the founders of p-adic mathematical physics. He has done tremendous efforts and energy in its worldwide dissemination. He is really one of driving forces of its development. His works received an international recognition.

The journal "p-Adic Numbers, Ultrametric Analysis and Applications" is indebted to B. Dragovich for his excellent work as deputy editor-in-chief.

Branko is not only a remarkable scientist but also a kind and a charming person. In any circumstances he is always being quiet, calm, wise and friendly. He has many friends from various countries.

We wish Branko a long, healthy, creative and happy life, with success in all his endeavors.

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