Serb. Astron. J. № 208 (2024), 1 - 15 **UDC 52-335.7 + 524.387**
DOI: https://doi.org/10.2298/SAJ2408001A Invited review

THE CRITICAL MASS RATIO FOR W UMA-TYPE CONTACT BINARY SYSTEMS

Bojan Arbutina[1] and **Surjit Wadhwa**[2]

[1] *Department of Astronomy, Faculty of Mathematics, University of Belgrade Studentski trg 16, 11000 Belgrade, Serbia*

E–mail: *arbo@astro.edu.rs*

[2] *School of Science, Western Sydney University, Locked Bag 1797, Penrith, NSW 2751, Australia*

(Received: March 31, 2024; Accepted: March 31, 2024)

SUMMARY: Contact binaries are close binary systems in which both components fill their inner Roche lobes so that the stars are in direct contact, and in potential mass and energy exchange. The most common such systems of low mass are the so-called W UMa-type. In the last few years, there has been a growing interest of the astronomical community in stellar mergers, primarily due to the detection of gravitational waves (mergers of black holes and neutron stars), but also because of an alternative model for the type Ia supernovae (merger of two white dwarfs), which are again particularly important in cosmology where they played a significant role in the discovery of dark energy and the accelerated expansion of the Universe. In that sense, contact systems of W UMa type with extremely low mass ratio are especially interesting because there are indications that, in their case too, stars can merge and possibly form fast-rotating stars such as FC Com stars and the blue-stragglers, and (luminous) red novae such as V1309 Sco. Namely, the previous theoretical research has shown that in the cases when the orbital angular momentum of the system is only about three times larger than the rotational angular momentum of the primary, a tidal Darwin's instability occurs, the components can no longer remain in synchronous rotation, orbit continue to shrink fast, and they finally merge into a single star. The above stability condition for contact systems can be linked to a specific critical mass ratio below which we expect a system to be unstable. We give an overview of this condition and show how it can be used to identify potential mergers. Finally, we discuss a number of known extreme mass ratio binaries from the literature and consider prospects for future research on this topic.

Key words. Binaries: close – Blue stragglers – Instabilities – Methods: analytical

1. INTRODUCTION

Contact systems represent close binary systems in which both components fill their inner Roche lobes so that the stars are in direct contact and in potential exchange of mass and energy. Although there are massive OB, even O-O contact binaries (Abdul-Masih et al. 2021, 2022), the most common such low-mass systems are the so-called stars of the W Ursae Majoris (W UMa) type systems with components of late spectral classes, which have a common convective envelope and approximately the same effective temperatures (Hilditch 2001).

In the last few years, special attention of astronomers has been attracted by stellar mergers, primarily due to the detection of gravitational waves (mergers of black holes and neutron stars) (Abbott et al. 2016, 2017), but also as an alternative model for type Ia supernovae (merger of two white dwarfs), which are again particularly significant in cosmology where they played an important role in the discovery of dark energy and the accelerated expansion of the Universe (Riess et al. 1998, Perlmutter et al. 1999). It was initially believed that contact W UMa-type binary systems would dominate the Galactic gravitational wave background at low frequencies (detectable by the future Laser Interferometer Space Antenna - LISA). However, it was realized later that it would most probably be completely dominated by detached and semidetached (AM CVn-type) white dwarf binary systems (see Postnov and Yungelson 2014, and references therein). However, contact systems of type W UMa with a low mass ratio are still particularly interesting because there are indications that in their case, a merger of stars can also occur and a possible formation of fast-rotating stars of type FK Com and the so-called "blues stragglers", and (luminous) red novae.

Namely, earlier theoretical research has shown that in the case when the orbital angular momentum is only about three times greater than the rotational angular momentum of the primary component, tidal instability occurs (Darwin 1879), the components can no longer remain in synchronous rotation, they rapidly spirally approach each other and finally merge into a single star. This stability condition for contact systems can be connected to the existence of a specific critical mass ratio below which we expect the system to be unstable.

This paper is organized as follows: in Section 2 we give a brief overview of W UMa-type binaries, review and reanalyze the condition for their stability (Subsection 2.1), and, finally, compare the theoretical results with the observational data, with a view to identifying potential merger candidates (Subsection 2.2); in Section 3 we discuss prospects for future research and in Section 4 we give a conclusion.

2. CONTACT BINARIES OF W UMA-TYPE

W UMa-type binary systems were named after its prototype W UMa, an eclipsing binary of spectral type F5 V discovered by Muller and Kempf (1903), with a mass ratio $q = 0.508$, and primary mass $M_1 = 1.14\ M_\odot$ (Gazeas et al. 2021a). W UMa binaries were defined as a class in the reviews of Binnendijk (1965, 1970, 1977) and Rucinski (1985a,b). These are stars of spectral type late F–K, which have a common convective envelope and nearly equal effective temperatures, although the mass of the primary component is usually significantly greater than that of the secondary (typically twice i.e. $q = M_2/M_1 \sim 0.5$, but up to ten times or more). Primary components in W UMa binaries seem to be normal main-sequence (MS) stars, while secondaries are oversized for their zero-age main-sequence (ZAMS) masses, and can be found left from the MS (Rucinski 1993, Hilditch 2001). The mass transfer in contact binaries, whether from primary to secondary or vice versa, does not seem to be that large (on short timescales, at least), but the energy i.e. luminosity transfer needs to be substantial, to equalize effective temperatures of quite different stars, perhaps involving differential rotation (Yakut and Eggleton 2005, Eggleton 2006).

There are two sub-types of W UMa binaries: A and W; the stars being classified as the former or the latter sub-type depending on whether the primary or the secondary is eclipsed during the primary (deeper) minimum, and consequently, whether the primary or the secondary has a slightly higher temperature. We thus arrive at another peculiarity observed in W subtype W UMa-type binaries, that the secondary, less massive star is hotter. In rare cases it seems, somewhat contradictory, that the secondary is slightly hotter in A sub-type systems (Gazeas et al. 2007, Alton and Stępień 2016). Spot presence and their specific locations may account for this A/W-subtype ambiguity (Alton and Stępień 2021). There are also some other suggested sub-types. Lucy and Wilson (1979) introduced the class of B-type systems which are systems in geometrical contact, but not in thermal contact and, therefore, there are large surface temperature differences between the components. H subtype (high mass ratio) systems were introduced by Csizmadia and Klagyivik (2004). Low mass contact binaries of W UMa–type also exhibit a short period cut-off at $P \sim 0.15 - 0.22$ days whose origin is still under debate (Rucinski 1992a, Stepien 2006b, Zhang and Qian 2020, Loukaidou et al. 2022, Zhang et al. 2023, Papageorgiou et al. 2023).

W UMa binaries are likely formed from detached systems of low mass, with orbital periods $P \lesssim 1$ day, which lose angular momentum through some mechanism, probably magnetized stellar wind. Magnetized wind, and generally magnetic activity, and presence of starspots i.e. O'Connell (1951) effect, are also characteristic of W UMa systems (Hilditch 2001). As the orbit shrinks, the primary will first touch its Roche lobe and a transfer relatively small amount of mass to the MS secondary before stars get into contact (see Fig. 1). The system can stay in contact for an uncertain time interval in the so-called thermal-relaxation-oscillations (TRO) regime (Flannery 1976, Lucy 1976). During TRO, the system should actually oscillate between contact and marginally detached configuration, remaining in quasi-equilibrium. Mass exchange first assumes the flow from the secondary to primary, leading to the orbit widening and loss of contact, but the primary would then expand, the mass transfer from primary to secondary via the Roche lobe overflow (RLOF) will lead to orbit shrinking, the contact is re-established, and everything repeats from the beginning (Rucinski 1993, Hilditch 2001). Alternatively, the mass transfer via RLOF with mass ratio reversal may have happened prior to the first contact, implying that present secondaries are in an advanced evolutionary stage (Stepien 2006a).

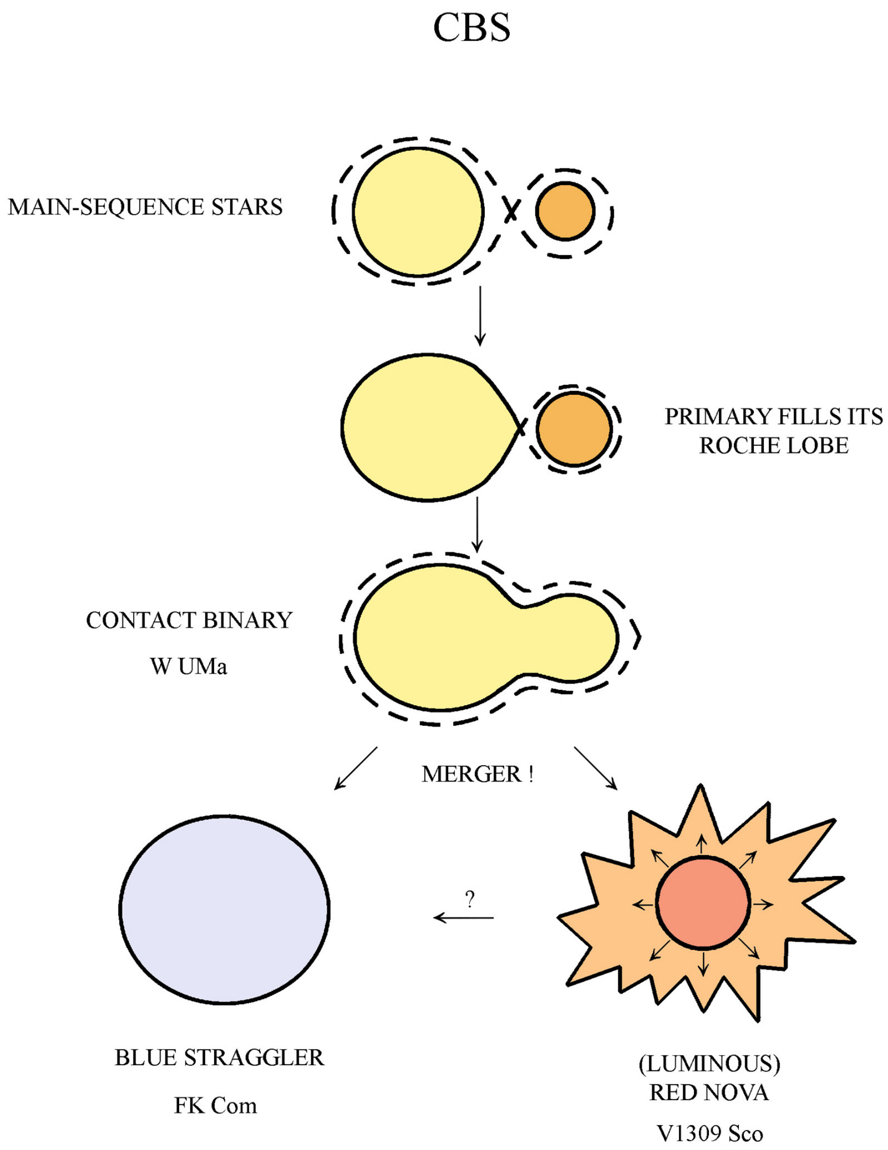

Fig. 1: A possible evolution of W UMa-type close binary systems (CBS) cartoon.

In any case, due to the angular momentum loss (AML), the stars would eventually merge, perhaps forming a rapidly rotating object such as blue stragglers. In favor of this scenario is a number of W UMa-type binary systems among blue stragglers in open and globular clusters (Kaluzny and Shara 1987). If a blue straggler is formed in a stellar merger, is its formation preceded by a red nova event such as V1309 Sco (Fig. 1, Ferreira et al. 2019)? Alternatively (or predominantly), blue stragglers in old open clusters may form via mass-transfer from an asymptotic giant branch (AGB) or red giant branch (RGB) companion (Leiner et al. 2019, Leiner and Geller 2021). There is also a possibility for direct collisions in high stellar concentration cores of globular clusters, i.e. close encounters of single stars that will end in a coalescence. Of course, it is possible that some of these interesting objects, particularly those in young stellar environments, do not originate in merger or collisions, but in starbursts or delayed star formation (Eggen and Iben 1989).

2.1. Stability criterion and the instability mass ratio

We have seen that the long-term dynamical evolution of W UMa binaries is driven presumably by AML. In close binary systems, tidal forces lead to synchronization and circularization of orbit. If the timescale for the synchronization is smaller than the AML timescale, the system will remain synchronized and the orbit will shrink until, at some critical separation, the instability sets in – the so-called secular, tidal, or Darwin (1879) instability. At this point, the rotational and orbital angular momentum become comparable ($J_{\rm orb} \approx 3J_{\rm spin}$, Hut 1980). The system can no longer stay synchronized, and since the angular momentum is still lost, the orbit will rapidly shrink until the final merger.

As we mentioned, the stability condition for contact systems can be linked to a minimum mass ratio below which we expect a system to be unstable (Rasio 1995, Li and Zhang 2006, Arbutina 2007, 2009a,b, Jiang et al. 2010, Wadhwa et al. 2021, Zhang 2024). Rasio (1995) showed that the minimum mass ratio would depend on the dimensionless gyration radius of the primary, defined through the moment of inertia of a star $I = k_1^2 M_1 R_1^2$. In other words, the stellar structure determines the gyro-radius that enters into the calculation of the minimum mass ratio. For a fully radiative primary, taken to be $n = 3$–polytrope, Rasio's expression $a/R_1 = k_1\sqrt{3(1+q)/q}$, for a system in marginal contact (R_1 taken to be the mean radius of the inner Roche lobe for the primary and $k_1^2 = 0.075$), gives $q_{\rm min} = 0.085$. Nevertheless, it was already known at the time about the extremely low mass ratio contact binary AW UMa with $q = 0.075$ (Rucinski 1992b). To place the AW UMa just at the stability boundary, it needed $k_1^2 \approx 0.06$, implying that its primary cannot have much of the convective envelope and must be slightly evolved (Rasio 1995). But, there were other extremely low mass ratio systems discovered, such as V857 Her with $q = 0.065$ Qian et al. (2006).

On theoretical grounds the contribution of the rotational angular momentum of the secondary and inclusion of the gyro-radius k_2 was considered by Li and Zhang (2006). Arbutina (2007), in addition to this, has taken into account the fact that the radii R_2 and R_1 in contact binaries are correlated, and through this correlation there was an additional dependence of the angular momentum on binary separation. Taking into account the structure of the primary components (deformation of the primary due to rotation and companion presence), Arbutina (2009a) included a nonzero quadruple moment in the calculation i.e. apsidal motion constant, which slightly improved the $q_{\rm min}$ value but the problem remained. However, the analysis of Arbutina (2009a) was performed for an idealized fully radiative primary, i.e. $n = 3$–polytrope, while it was known that the evolved MS stars with $M_1 \gtrsim 1\ M_\odot$ can have a lower gyro-radius (Li and Zhang 2006, Jiang et al. 2010). Jiang et al. (2010) reanalyzed the minimum mass ratio by considering the structure of MS stars using Eggleton's stellar evolution code (Eggleton 1971, 1972, Eggleton et al. 1973, Eggleton and Kiseleva-Eggleton 2002), emphasizing the importance of the primary's gyro-radius k_1. By analyzing statistically empirical relations for deep, low mass ratio contact binaries, Yang and Qian (2015) concluded that $q_{\rm min}$ could be as low as 0.044. Wadhwa et al. (2021) constructed a $k_1 - M_1$ relation for ZAMS stars based on Landin et al. (2009) calculations for rotationally and tidally distorted components in close binaries.

In the following lines we shall repeat the stability analysis of Arbutina (2007) and Wadhwa et al. (2021) by assuming a certain dependence of filling factor on stellar volume radii, fit the gyro-radius–mass dependence for ZAMS primaries and derive an improved theoretical stability condition. We start from the total angular momentum of a binary

$$J_{\rm tot} = J_{\rm orb} + J_{\rm spin} = J_{\rm orb} + J_1 + J_2, \qquad (1)$$

where J_1 and J_2 are spin angular momenta of the components. The orbital angular momentum of a binary can be written as

$$J_{\rm orb} = \mu a^2 \Omega = \frac{q\sqrt{GM^3 a}}{(1+q)^2}, \qquad (2)$$

where the reduced mass is $\mu = M_1 M_2/M$, the total mass is $M = M_1 + M_2$, the mass ratio $q = M_2/M_1 < 1$, and M_1 and M_2 are masses of the primary and secondary component, respectively. $\Omega = \sqrt{GM/a^3}$ is the (Keplerian) orbital angular velocity, while a is binary separation. Assuming synchronization, the spin angular momentum of a binary is

$$J_{\rm spin} = k_1^2 M_1 R_1^2 \Omega + k_2^2 M_2 R_2^2 \Omega, \qquad (3)$$

where R_1 and R_2 are taken to be the volume radii (see Mochnacki 1984), and k_1 and k_2 are gyro-radii.

The overcontact degree for a contact binary is defined as

$$f = \frac{\Phi_{\rm eff} - \Phi_{\rm IL}}{\Phi_{\rm OL} - \Phi_{\rm IL}}. \qquad (4)$$

Value $f = 0$ corresponds to marginal contact (components reaching the L1 point), while $f = 1$ corresponds to full overcontact configuration (the secondary reaching the L2 point). Yakut and Eggleton (2005) adopted a logarithmic, Arbutina (2007) adopted linear dependence of $\Phi_{\rm eff}$ on R, while Arbutina (2009a) assumed $\Phi_{\rm eff} \propto 1/R$. We adopt the latter dependence, which seems reasonable because, if $\Phi \propto 1/r$, the effective potential should be well represented by a similar dependence on volume ("effective") radius. Nevertheless, the exact dependence is not that important in the narrow range $f = 0 - 1$ where all approximations give a similar accuracy.

Thereby, from Eq. (4) we have

$$f \approx \frac{1/R - 1/R_{\rm IL}}{1/R_{\rm OL} - 1/R_{\rm IL}}, \qquad (5)$$

where volume radii for the inner Roche lobes, touching at L1, are approximately Eggleton (1983)

$$\frac{R_{{\rm IL}i}}{a} = \begin{cases} \frac{0.49q^{-2/3}}{0.6q^{-2/3}+\ln(1+q^{-1/3})}, & i = 1 \\ \frac{0.49q^{2/3}}{0.6q^{2/3}+\ln(1+q^{1/3})}, & i = 2, \end{cases} \qquad (6)$$

while for the volume radii of the outer Roche lobes we suggest

$$\frac{R_{\rm OL1}}{a} = \frac{0.49q^{-2/3}\big(\cosh(1.15q^{2/5})\big)^{1/2}}{0.6q^{-2/3} + \ln(1+q^{-1/3})}, \quad (7)$$

$$\frac{R_{\rm OL2}}{a} = \frac{0.49q^{2/3}\big(1 - q^{1/4}\tanh^2(1.15q^{-1/5})\big)^{-1/4}}{0.6q^{2/3} + \ln(1+q^{1/3})}.$$

The latter are defined as the radii of the spheres, each being of the same volume as the volume of the respective figure obtained by cutting the equipotential surface passing through the L2 point by a plane through the L1 point which is perpendicular to the line of centers. Formulae in Eq. (7) are slightly better approximations than those given by Yakut and Eggleton (2005), accurate to less than 1 per cent when compared to the Mochnacki (1984) tables (Fig. 2).

As the component's surfaces in the contact system are at the same potential (the same f), the stellar radii are correlated, and by combining the above equations, one obtains

$$R_2 = \frac{R_{\rm IL2}R_{\rm OL2}}{fR_{\rm IL2} + (1-f)R_{\rm OL2}}, \quad (8)$$

where

$$f = \frac{1/R_1 - 1/R_{\rm IL1}}{1/R_{\rm OL1} - 1/R_{\rm IL1}}. \quad (9)$$

From the instability condition $\frac{{\rm d}J_{\rm tot}}{{\rm d}(a/R_1)} = 0$ one finds the equation for the critical separation

$$\left(\frac{a_{\rm inst}}{R_1}\right)^2 = \frac{1+q}{q}\left[3k_1^2 - qk_2^2\left(\frac{R_2}{R_1}\right)^2\left(1 - 4\frac{R_2}{R_1}S\right)\right], \quad (10)$$

where

$$S(q) = \frac{1/R_{\rm OL2} - 1/R_{\rm IL2}}{1/R_{\rm OL1} - 1/R_{\rm IL1}}. \quad (11)$$

In a situation when the secondary (i.e. its angular momentum) has been neglected ($k_2 = 0$), the instability equation is reduced to

$$\frac{a_{\rm inst}}{R_1} = k_1\sqrt{\frac{3(1+q)}{q}}, \quad (12)$$

which is the result found in Rasio (1995), that we quoted above.

By transforming Eq. (9) to obtain the equation for the primary equivalent to Eq. (8)

$$R_1 = \frac{R_{\rm IL1}R_{\rm OL1}}{fR_{\rm IL1} + (1-f)R_{\rm OL1}}, \quad (13)$$

we obtain from the instability equation an algebraic equation for the instability mass ratio that depends on k_1^2, k_2^2, and f. Both above expressions for R_1 and R_2 are accurate to less than 1 per cent when compared to the data in Mochnacki (1984). The radii ratio in Eq. (10) is explicitly

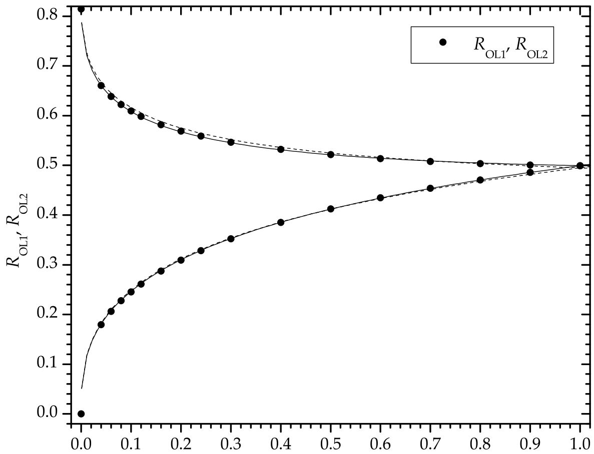

Fig. 2: Mean radii for the outer Roche lobes $R_{\rm OL1}$ and $R_{\rm OL2}$. Filled circles are numerical data from the Mochnacki (1984) tables, dashed curves are approximations given by Yakut and Eggleton (2005), while solid curves represent our approximations from Eq. (7).

$$\frac{R_2}{R_1} = \frac{f\big(\cosh(\frac{23}{20}q^{2/5})\big)^{-1/2} + 1 - f}{f\big(1 - q^{1/4}\tanh^2(\frac{23}{20}q^{1/5})\big)^{1/4} + 1 - f} \cdot \frac{1 + \frac{5}{3}q^{2/3}\ln(1+q^{-1/3})}{1 + \frac{5}{3}q^{-2/3}\ln(1+q^{1/3})} \quad (14)$$

To lesser accuracy this ratio for a fixed filling factor (or for all f) could be represented by a power-law $R_2/R_1 = q^p$ (see Fig. 3).

Although it is clear from papers by Jiang et al. (2010) and Wadhwa et al. (2021) that the gyration radius k_1 depends on M_1 and has minimum at $\sim 1.5M_\odot$ for ZAMS models (e.g. Landin et al. 2009), we shall show this explicitly by fitting the $k_1^2 = k_1^2(M_1)$ relation (Fig. 4). Since the Landin et al. (2009) calculations for the binary model assumed $q = 1$, and we are dealing with low-mass ratio systems, we fitted both – the binary models data that include rotational and tidal effects, and isolated star models ($q = 0$, $\Omega = 0$). We assumed a Gaussian + linear dependence of k_1^2:

$$k_1^2 = Ce^{-((M_1-m)/s)^2} + aM_1 + b, \quad (15)$$

with best fit parameters given in Table 1 (M_1 is in Solar mass units).

By using the last equation and assuming a fully convective secondary (n=1.5–polytrope) with $k_2^2 = 0.205$, one can find the minimum mass ratio

$$q_{\rm min} = 0.042 - 0.044, \quad (16)$$

for the filling factor $f = 0 - 1$. One must bear in mind that this is a global minimum, and that $q_{\rm inst}$

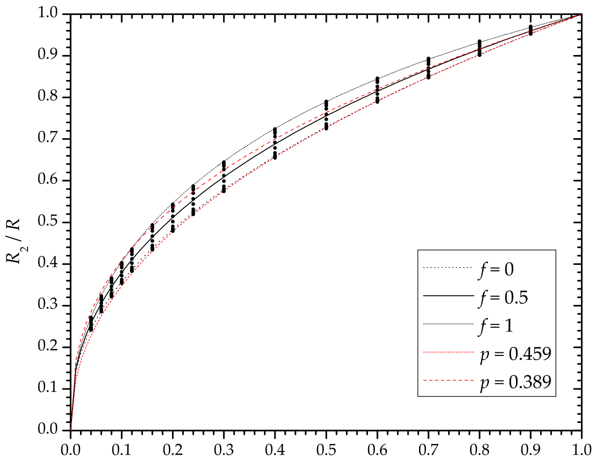

Fig. 3: The numerical data for the ratio R_2/R_1 from Yakut and Eggleton (2005) and our approximations. The dotted curve represents a power-law fits $R_2/R_1 \approx q^p$ to the observational data ($p = 0.459$, Kuiper 1941, Csizmadia and Klagyivik 2004) ($p = 0.389$, Poro et al. 2022).

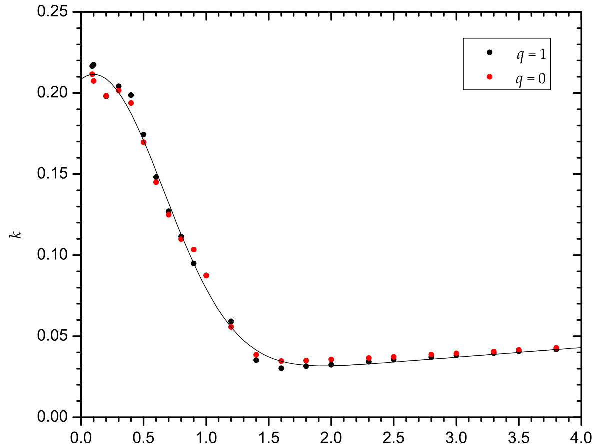

Fig. 4: The data (Landin et al. 2009) and $k_1^2 = k_1^2(M_1)$ relation fit.

Table 1: Parameters in the gyro-radius–primary's mass relation $k_1^2(M_1) = Ce^{-((M_1-m)/s)^2} + aM_1 + b$.

Parameter	
C	0.192 ± 0.006
m	0.09 ± 0.03
s	0.81 ± 0.04
a	0.006 ± 0.002
b	0.019 ± 0.005

that is of practical use, is different for each particular binary (Wadhwa et al. 2023a).

The instability mass ratio versus primary mass for $k_2^2 = 0.205$ and $f = 0 - 1$ is shown in Fig. 5. The data are from Latković et al. (2021). Extreme and low mass ratio contact binaries that we consider as possible merger candidates are listed in Table 2 and included in Fig. 5 as well. Systems listed by Li et al. (2021) that we did not include in the table since they are probably not merger candidates, based on our analysis (being relatively massive), are: V857 Her, M4 V53, V870 Ara, KR Com, FP Boo, KIC 11097678, XX Sex and AW Crv. The situation is probably similar with AW UMa ($q = 0.076$) (Eaton 2016), VESPA V22 ($q = 0.079$) (Popov and Petrov 2022), GSC 02265-01456 ($q = 0.087$) (Guo et al. 2015), NW Aps ($q = 0.10$) and AL Lep ($q = 0.12$) (Wadhwa 2005). Recently, Christopoulou et al. (2022), Liu et al. (2023), Wadhwa et al. (2023a,b), Lalounta et al. (2024) found a number of low-mass ratio W UMa-type systems, none of them, however, satisfying our instability criterion, although some are close. The majority of systems that we included in Table 2 also seem to be stable for now. Only nine fulfill the criterion for instability, having $q < q_{\rm inst}$. We discuss them and some other interesting systems in more detail in the following subsection.

2.2. Interesting systems

2.2.1. FK Com

HD 117555 was recognized to be a rapidly rotating G-type star by Merrill (1948). Chugainov (1966) showed this star to be (micro)variable with small brightness variation of ~ 0.1 magnitude and period of $P \approx 2.4$ days, after which it was designated FK Comae Berenices (FK Com). FK Com was soon suggested to be the prototype of a new class of variables, including UZ Lib and V1794 Cygni (Bopp and Stencel 1981, Bopp and Rucinski 1981, Rucinski 1981). It is a giant star (G2 III – G7 III) with unusually fast rotation, with $v \sin i \approx 160$ km/s, large cool spots and chromospheric activity, as well as long term variability (Panov and Dimitrov 2007). FK Com shows a variable radio, X-rays, UV and Hα emission (Kjurkchieva and Marchev 2005).

In some characteristics (enhanced chromospheric, transition region, and coronal emission) the FK Com–type stars are similar to RS CVn–type (RS Canum Venaticorum) systems (Montesinos et al. 1988). Nevertheless, FK Com displays a lack of an observable radial velocity variation due to binarity and broad Hα emission line with a strongly variable profile. Its binary nature is not completely excluded. Walter and Basri (1982) suggested that the giant may be accreting mass from a small unseen companion, but since the attempts to reveal the companion were unsuccessful, it would have been indeed a very small, low-mass star (McCarthy and Ramsey 1984). The main arguments for the binary model may be the long-term stability of the brightness variations, since dark starspots appearing always at the same locations

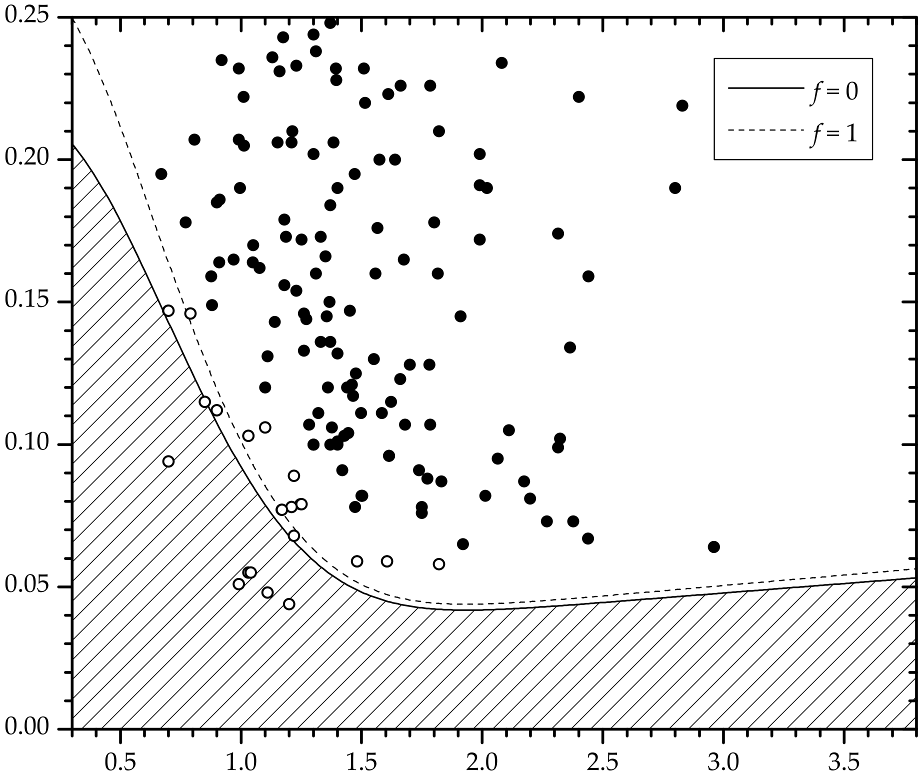

Fig. 5: The instability mass ratio versus primary mass. The data are from Latković et al. (2021) (filled circles) and Table 2 (open circles).

i.e. longitudes in the single star model, is not what one would expect (Rucinski 1991). However, if FK Com was a low–mass ratio system it may be that the stars have already merged i.e. that this interesting rapidly rotating giant is actually the result of merger of a W UMa-type contact binary (Ayres et al. 2016, and references therein).

2.2.2. *AW UMa*

Paczyncki's star AW UMa (BD +30°2163) was discovered in 1964 as a W UMa–type eclipsing binary with a period of about $P \approx 0.44$ days (Paczynski 1964). This was the first extremely low mass ratio contact binary with $q = 0.075$ (Rucinski 1992b) and a record holder for decades. However, Pribulla and Rucinski (2008) found a higher mass ratio $q \sim 0.1$ and they suggested that AW UMa may not be a contact binary after all. In their new model, AW UMa was a detached system, but with equatorial disk (belt) encompassing both components. This view was further supported by high-resolution spectroscopic observations by Rucinski (2015), who suggested that AW UMa is a very tight, semi-detached binary with a mass transfer from the more massive to the less massive component. Eaton (2016) criticised the latter model, reanalyzed the system, and provided an alternative solution, among others, in which AW UMa is a contact binary with $q = 0.076$, having polar spots. The latter author concluded that a better approach to explain the line profiles would be to consider a differential rotation of both components (Eaton 2016). The author, however, did not provide any direct evidence for the existence of spots and did not discuss the distinct possibility of the spotted solution being non-unique, and a potential existence of other solutions with significantly different geometric parameters (Maceroni and van't Veer 1993, Eker 1999).

AW UMa shows a steady period decrease $\dot{P} = -0.145 \times 10^{-7}$ (Wilson 2008), but no actual signs of instability. From the primary temperature $T_1 = 7410$ K, we estimate its mass $M_1 \approx 1.75\ M_\odot$ (Pecaut et al. 2012, Pecaut and Mamajek 2013). Pribulla and Rucinski (2008) find $M_1 \sim 1.5\ M_\odot$ but for a lower temperature $T_1 = 6980$ K. In any case, a higher mass of the primary, in comparison to the Solar value, suggests that the system is not unstable, despite low mass ratio (see Fig. 5).

2.2.3. *V1309 Sco*

V1309 Scorpii (V1309 Sco) was discovered as a nova in 2008 by Nakano et al. (2008). Soon it became clear that this was not a typical classical nova, but the so-called red nova (Mason et al. 2010), and was later characterised as a Rosetta stone of contact binary

Table 2: Extreme and low mass ratio contact binaries – potential merger candidates. Instability mass ratio is given only for systems with $q < q_{\rm inst}$.

Name		Mass			Temperatures			Ref.
	q	M_1 $[M_\odot]$	f	i [°]	T_1 [K]	T_2 [K]	$q_{\rm inst}$	
V1187 Her	0.044	1.20[b]	0.84	66.7	6250	6682	0.072	(1)
TYC 4002-2628-1	0.048	1.11	0.35	69.7	6032	6151	0.096	(2)
WISE J185503.7	0.051	0.99	0.16	70.7	5747	5827	0.095	(3)
WISE J141530.7	0.055	1.04	0.64	77.4	5890	5966	0.091	(4)
VSX J082700.8	0.055	1.03[b]	0.19	68.7	5870	5728	0.089	(5)
IP Lyn[a]	0.058	1.82	0.22	76.8	6680	6180	–	(6)
KIC 4244929	0.059	1.48	0.81	70.6	6857	6867	–	(7)
KIC 9151972	0.059	1.61	0.76	70.1	6040	5982	–	(7)
ASAS J083241+2332.4	0.068	1.22	0.69	82.7	6300	6672	0.069	(8)
NSVS 2569022	0.077	1.17	0.01	76.3	6100	6100	–	(9)
ZZ PsA	0.078	1.21	0.97	72.2	6514	6703	–	(10)
SX Crv	0.079[c]	1.25	0.27	61.2	6340	6160	–	(11)
1SWASP J132829	0.089	1.22[b]	0.70	81.5	6300	6319	–	(5)
V1309 Sco[d]	0.094	0.7?[e]	0.89	73.4	4500	4354	0.161	(12)
ASAS J165139+2255.7	0.103	1.03	0.60	78.1	5370	5394	–	(13)
ASAS J082243+1927.0	0.106	1.10	0.78	76.6	5960	6078	–	(14)
V1222 Tau	0.112	0.90	0.54	80.2	5425	5600	0.113	(15)
GSC 02800-01387	0.115	0.85	0.63	74.6	5684	5846	0.124	(16)
NSVS 1917038	0.146	0.79[b]	0.04	73.8	4869	5074	–	(17)
NSVS 4316778	0.147	0.70	0.00	75.6	5960	6100	–	(18)

[a]No–spots solution. [b]Main-sequence mass based on temperature from the 2022 updated version of tables by Pecaut et al. (2012) and Pecaut and Mamajek (2013).* [c]Spectroscopic mass ratio. [d]Parameters before merger. [e]Assigned mass for a fictitious main-sequence doppelganger.

References: (1) Caton et al. (2019), (2) Guo et al. (2022), (3) Guo et al. (2023b), (4) Guo et al. (2023a), (5) Li et al. (2021), (6) Yin et al. (2023), (7) Şenavcı et al. (2016), (8) Sriram et al. (2016), (9) Kjurkchieva et al. (2018), (10) Wadhwa et al. (2021), (11) Zola et al. (2004), (12) Zhu et al. (2016), (13) Alton (2018), (14) Kandulapati et al. (2015), (15) Liu et al. (2015), (16) Popov and Petrov (2022), (17) Guo et al. (2020), (18) Kjurkchieva et al. (2020).

*http://www.pas.rochester.edu/~emamajek/EEM_dwarf_UBVIJHK_colors_Teff.txt

mergers (Tylenda et al. 2011, Tylenda and Kamiński 2016).

Optical Gravitational Lensing Experiment (OGLE) observations exist for the star in the period 2001–2008, when the outburst happened (Udalski 2003). Tylenda et al. (2011) found V1309 Sco progenitor to be a K1-3 III giant, in a system with initial period of about 1.44 days and with exponential decay

$$P = P_0 \exp(\tau/(t - t_0)), \tag{17}$$

where t is time in Julian Dates (JD), $t_0 = 2455233.5$ and $\tau = 15.29$ and $P_0 = 1.4456$ days. It may be that V1309 Sco was a contact binary of W UMa type, but as stated earlier, these systems generally have primaries that can be regarded as MS stars, and orbital periods which are typically less than a day, V1309 Sco then, perhaps being at the long period cut-off (Rucinski 1998).

Taking into account the observed characteristics above, Stępień (2011) concluded that V1309 Sco was different from W UMa-type stars – the primary being a giant that recently filled its Roche lobe, and that the contact phase was very short, ending in a merger. Nandez et al. (2014) modelled the system as a contact binary having a sub-giant primary with $M_1 \approx 1.52\ M_\odot$ and mass ratio $q = 0.105$. Tylenda et al. (2011) assumed a lower mass $\sim 1\ M_\odot$. Since our analysis is for the MS stars, to see where V1309 Sco would be on our $q - M_1$ plot (Fig. 5), we assign $M_1 \approx 0.7\ M_\odot$ to the primary, which is the MS mass corresponding to K-type star with $T_1 = 4500$ K. Binaries with a less massive primary should not reach RLOF, and, consequently, a contact configuration within the age of the Universe (Stepien 2006b). The mass ratio $q = 0.094$ (Zhu et al. 2016) of the assigned primary mass would definitely put V1309 Sco in the instability region. For an evolved primary, to push the system at the edge of stability $q_{\rm inst} \approx q = 0.094$ one would need a gyro-radius $k_1^2 \sim 0.073 - 0.081$ (see Fig. 6).

2.2.4. V857 Her

V857 Herculis (V857 Her) is an extreme mass ratio contact binary with a period of $P \approx 0.38$ days. Qian et al. (2005) report the photometric analysis of V857 Her and derive a mass ratio $q = 0.065$ and high fillout of 83.8%. They find a weak evidence that the orbital period may show a continuous increase at a rate of $\dot{P} = 2.9 \times 10^{-7}$ days/yr. The authors did not provide an estimate of the mass of the primary. Using various published empiric relations such as the absolute magnitude estimate of the primary from the secondary eclipse (Wadhwa et al. 2021) and mass-period relation (Yang and Qian 2015), we estimate the mass of the primary to be $(1.3 - 1.5)\ M_\odot$. Adopting the mean value of $M_1 = 1.4\ M_\odot$, we would consider V857 Her to be stable. Thus, the situation with this system may be similar to the AW UMa case – an extremely low mass ratio system, but with relatively massive primary. In addition, it is possible that the light curve of V857 Her is influenced by presence of a hot sub-dwarf (Pribulla et al. 2009).

2.2.5. V1187 Her

V1187 Herculis (V1187 Her) was discovered by Robotic Optical Transient Search Experiment I (ROTSE I) and designated as ROTSE-1 J162919.83+353959.2 (Akerlof et al. 2000). It was classified as an EW variable with amplitude of about 0.2 magnitudes and period $P \approx 0.31$ days. With $q = 0.044$, V1187 Her currently holds the record for the most extreme low-mass ratio contact binary system (Caton et al. 2019). The system exhibits a period change at rate $\dot{P} = -1.5 \times 10^{-7}$ days/year.

Although the original authors do not provide a mass estimate of the primary, the spectroscopic classification and observational evidence would suggest the mass of the primary to be in the order of $M_1 = (1.1 - 1.2)\ M_\odot$. The system would undoubtedly be classified as unstable at this estimate ($q_{\rm inst} = 0.072$ for $M_1 = 1.2$, and $f = 0.84$). There is some evidence, however, suggesting that the system is contaminated by a significant third light, with the most recent estimates of the mass ratio of the system as high as 0.16 (Cook et al. 2022, Cook and Kobulnicky 2024).

2.2.6. TYC 4002-2628-1

TYC 4002-2628-1 (CzeV710, WISE J230927.8 +545123) was discovered as an EW variable with a period $P \approx 0.37$ days by Pintr (2015) (see also Skarka et al. 2017). The photometric observation and light-curve analysis by Guo et al. (2022) find TYC 4002-2628-1 to be an extreme low-mass ratio system with $q = 0.048$. The ephemeris shows a secular period increase of $\dot{P} = 1.62 \times 10^{-5}$ days/year. This high long-term period increase was suggested by the authors to be a consequence of mass transfer from the secondary to the primary star. For the assumed primary mass $M_1 = 1.11\ M_\odot$ and fillout $f = 0.35$, the instability mass ratio is $q_{\rm inst} = 0.096$. For the above system parameters, TYC 4002-2628-1 is thus significantly below the stability limit and should be unstable.

2.2.7. WISE J185503.7+592234

WISE J185503.7+592234 (ASASSN-V J185503.69 +611804.1, ZTF J185503.69+611804.2) was discovered by Zwicky Transient Facility (ZTF) (Chen et al. 2020). Guo et al. (2023b) recently observed and analyzed the system, finding it to be a particularly low mass ratio ($q \approx 0.051$) contact binary approaching merger. The period of the binary is $P \approx 0.28$ days with secular period decrease of $\dot{P} = -2.24 \times 10^{-7}$ days/year. The authors interpret this period change by mass transfer from the primary to the secondary, leading to an even smaller mass ratio, deeper contact, and eventual coalescence. For the estimated primary mass $M_1 \approx 0.99\ M_\odot$ and the fillout $f = 0.16$, the mass ratio is already significantly below the critical mass ratio $q_{\rm inst} = 0.095$ and the system should be unstable.

2.2.8. WISE J141530.7+592234

WISE J141530.7+592234 (ASASSN-V J141530.72 +592234.6, ZTF J141530.72+592234.3) was recognized as a contact binary with a low amplitude of ~ 0.2 magnitudes and a short period $P \approx 0.34$ days in Wide-field Infrared Survey Explorer (WISE) Catalog of Periodic Variable Stars (Chen et al. 2018) and ZTF (Chen et al. 2020). Guo et al. (2023a) find the system to be an extreme low-mass ratio binary with $q \approx 0.055$. The system displays significant light curve variations and O'Connell (1951) effect reversal. The period increase rate $\dot{P} = 3.90 \times 10^{-7}$ days/year is tentatively explained by mass transfer from the low mass secondary to the more massive primary component. For the estimated primary mass $M_1 = 1.04\ M_\odot$ and the filling factor $f = 0.64$ for a hot spot solution, the critical mass ratio is $q_{\rm inst} = 0.091$, making the system clearly unstable. This conclusion would not change for cool spot and no-spot solutions.

2.2.9. VSX J082700.8+462850

VSX J082700.8+462850 (VSX J082700) and 1SWASP J132829.37 +555246.1 are two extreme low mass ratio contact binaries analyzed by Li et al. (2021). While the latter has a higher mass ratio and probably more massive primary, our analysis suggests that the former should be unstable.

VSX J082700 was first classified as an EW variable by Srdoc (2010). The photometric light curve solution of the system provided by Li et al. (2021) suggests a mass ratio $q = 0.055$. The system has a period $P \approx 0.28$ that shows a decrease at the rate $\dot{P} = -9.52 \times 10^{-7}$ days/year. Based on photometric, color and empiric relationships, the estimated mass of the primary ranges from 1.03 $M_\odot$ to 1.15 $M_\odot$. For the primary mass $M_1 = 1.03$ $M_\odot$, which is the MS mass corresponding to $T_1 = 5870$ K, and $f = 0.19$, the instability mass ratio is $q_{\rm inst} = 0.089$. This places the system in the unstable category, similarly to V1187 Her. We find no evidence of blending from a nearby star. However, as this is the case with V1187 Her, this system has a very low inclination and a contributing third light therefore requires further investigation.

2.2.10. ASAS J083241+2332.4

ASAS J083241+2332.4 (NSVS 7399728, GSC 01941-02356) was observed with Kilodegree Extremely Little Telescope (KELT) by Pepper et al. (2008), classified as EB, and designated KP301148. Sriram et al. (2016) photometrically observed and analyzed the system and found it to be an extreme low-mass ratio contact binary. The system has a period $P \approx 0.31$ days with a secular increase at the rate of $\dot{P} \sim 0.0765$ sec/year, and sinusoidal modulation with a period of ~ 8.25 years, possibly due to the presence of a third body. The latter authors found the photometric mass ratio $q = 0.068$ for a solution with hot spot. The cool spot and no-spot solutions have slightly lower mass ratios, 0.065 and 0.067, respectively. Taking for the mass of the primary $M_1 = 1.22$, and filling factor $f = 0.69$ for the hot spot solution, we found the critical mass ratio $q_{\rm inst} = 0.069$. This makes the system unstable, but barely. A lower filling factor $f \approx 0$ and a higher primary mass would make it stable.

2.2.11. NSVS 2569022

Gettel et al. (2006) classified NSVS 2569022 as a variable of the EW type with period $P \approx 0.29$ days and amplitude of about ~ 0.2 magnitudes. Kjurkchieva et al. (2018) provided a photometric solution for the light curve with the estimated mass ratio 0.077. It is difficult to estimate the mass of the primary component due to unavailability of the GAIA distance estimate and no high cadence photometry in the V band. The reported mass $1.17 M_\odot$ of the primary, based on the period-mass relationship, would place the system near the instability boundary, but it would remain stable ($q_{\rm inst} = 0.071$). Although Cook and Kobulnicky (2024) do not provide an estimate of the mass ratio for the system, they state that the system contains a third light far more extreme than previously thought and does not have an extreme low mass ratio.

2.2.12. ZZ PsA

ZZ Piscis Austrinus (ZZ PsA) is a neglected bright southern contact binary recognized as a variable in 1967 (Strohmeier 1967), rediscovered by Demartino et al. (1996), and designated NSV 13890. The light curve was analysed by Wadhwa (2006), who found the mass ratio $q = 0.080$ and fillout of 90%. The system has a period $P \approx 0.38$ days. No period change was reported, owing to the lack of observations. The system was recently analyzed by Wadhwa et al. (2021) reporting the mass ratio of 0.078 with higher fillout and estimated mass $M_1 = 1.21$ $M_\odot$ of the primary, based on the apparent magnitude of the secondary. For the above primary mass and filling factor $f = 0.97$, the critical mass ratio for ZZ PsA is $q_{\rm inst} = 0.072$. This makes the system stable, but close to the instability region in Fig. 5.

2.2.13. V1222 Tau

V1222 Tauri (V1222 Tau, GSC 00650-00769) is an ignored low-mass ratio contact binary found by Bernasconi and Behrend (2002). The system has a period $P \approx 0.29$ days, with a possible secular increase rate of $\dot{P} = 8.19 \times 10^{-6}$ days/year (Liu et al. 2015). Liu et al. (2015) provides both the unspotted and spotted solution for the light curve. Adopting the spotted solution with $q = 0.112$, the reported mass of the primary $M_1 = 0.9$ $M_\odot$ and $f = 0.54$, we find the critical mass ratio $q_{\rm inst} = 0.113$, making the system marginally unstable. For the unspotted solution, the authors find an even lower mass ratio ($q = 0.104$) and higher fillout ($f = 0.58$), which does not change much $q_{\rm inst} = 0.114$ so the system remains unstable for these parameter values.

2.2.14. GSC 02800-01387

GSC 02800-01387 (VSX J011323.6+374319) was discovered as an EW variable by de Miguel (2010). It has a period $P \approx 0.3$ days. Popov and Petrov (2022) observed and provided light-curve fitting parameters for this and other three systems. They actually found VESPA V22 (Quadri et al. 2017) to be the most extreme mass ratio system of four targets, with $q = 0.079$, but with the primary mass $M_1 = 1.99$ $M_\odot$ it falls in the stable region. The authors provide a spotted light curve solution for GSC 02800-01387 with an estimated mass ratio $q = 0.115$ and estimated mass of the primary $M_1 = 0.85$ $M_\odot$. For this primary mass and filling factor $f = 0.63$ the critical mass ratio is $q_{\rm inst} = 0.124$. Apart from concerns regarding the non-uniqueness of spotted solutions our analysis shows this system to be unstable for the current parameter values.

2.2.15. NSVS 1917038

NSVS 1917038 is discovered as a low mass ratio binary ($q = 0.146$), with period $P \approx 0.32$ days and an unusually shallow contact degree of 4 per cent (Guo et al. 2020). We could not find much additional data for this system and assumed $M_1 = 0.79$ $M_\odot$ which is the MS mass corresponding to $T_1 = 4870$ K. For this primary mass and $f = 0.04$, the critical mass ratio is

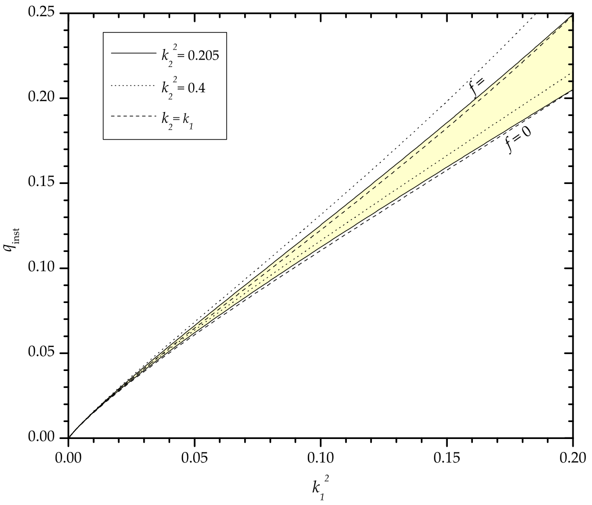

Fig. 6: The instability mass ratio, according to Eq. (10) versus the primary's dimensionless gyration radius, with three cases assumed for the secondary: n=1.5–polytrope with $k_2^2 = 0.205$, homogeneous sphere with $k_2^2 = \frac{2}{5}$, and $k_2 = k_1$. The lower curves correspond to the filling factor $f = 0$, while the upper curves correspond to $f = 1$.

$q_{\rm inst} = 0.127$, which makes the system stable. This conclusion would change if q and/or M_1 were slightly lower, and $f \lesssim 1$.

2.2.16. NSVS 4316778

NSVS 4316778 is an eclipsing binary with a period $P \approx 0.26$ days and an amplitude of about ~ 0.3 magnitudes (Woźniak et al. 2004). Kjurkchieva et al. (2020) performed a light curve analysis and found the system to be basically in marginal contact, with photometric mass ratio $q = 0.147$ and primary mass $M_1 \approx 0.7$. Both components of NSVS 4316778 seem to be oversized, greatly overluminous, and hotter when compared with MS stars of the same masses. Although system shows total eclipses, allowing for better parameter constraints, only a spotted solution is provided which makes the values somewhat unreliable. For the above parameters, assuming an MS primary, the critical mass ratio for NSVS 4316778 is $q_{\rm inst} = 0.143$, which makes it stable. However, a greater fillout $f \lesssim 1$ would place the system below the stability limit. The case of NSVS 4316778, as well as NSVS 1917038, demonstrates how a relatively high mass ratio $q \sim 0.15$ in combination with low primary mass could make systems potentially unstable.

3. PROSPECTS FOR FUTURE RESEARCH

After the review of systems in the preceding Section, an obvious question arises as to why there is no more direct evidence of instability for binaries with $q < q_{\rm inst}$. Concerning the observational data on the low mass ratio for the W UMa-type binaries, one can hope that the future research provides more reliable system parameters, the primary's mass M_1, and mass ratio q in particular. The high resolution spectroscopy would be needed very much in this context. One should closely monitor the systems that are considered unstable and those at the border of instability region (in Fig. 5), e.g. search for a rapid period change (Hong et al. 2024), unusual brightness variation, and other signs of instability. Comparisons with observational characteristics of V1309 Sco and other possible Galactic red novae such as V4332 Sgr and V1148 Sgr (Martini et al. 1999, Pastorello et al. 2019, Bond et al. 2022, and references therein) can be extremely useful.

A theoretically derived critical mass ratio can be further improved by considering metallicity dependence. This has already been started by Wadhwa et al. (2024). If the age of a system could be estimated more reliably, the evolutionary models for the primary, i.e. its structure that determines a gyro-radius k_1, could also come into play (Jiang et al. 2010). Concerning the secondary's gyro-radius, it is questionable whether k_2 for the n=1.5–polytrope should be used, and could a more adequate model be constructed for an oversized, tidally and rotationally distorted star. Li et al. (2005) argued that the efficient energy transfer would decrease the gyro-radius of the secondary and that the value of k_2 would not be much different from k_1, despite a significant difference in stellar masses. It is worth noting that for all the systems in Table 2 with mass ratio $q \leq 0.055$, the secondary's mass is less than the minimum mass for hydrogen fusion in isolated ZAMS stars, $M_2 < 0.08\ M_\odot$.

From a purely mathematical perspective, one could notice that in the derivation of the criterion for tidal instability in Subsection 2.1, the total mass and mass ratio are treated as constants (we are basically applying the criterion for their current values), while this is surely physically unjustified – a more careful analysis should somehow try to account for mass transfer and/or mass loss. Observations suggest that the mass ratio decreases over the course of time, perhaps not linearly but oscillatory, due to the TRO cycles, but the W UMa binaries do pile up at small q values (Pešta and Pejcha 2023). It is also possible that low-mass systems may reach full overcontact $f = 1$, and that the instability driving mechanism could then be the mass loss and AML through L2 (Webbink 1976, Tylenda et al. 2011, Hubová and Pejcha 2019). This is a situation that should further be investigated, perhaps leading to a different criterion.

One interesting extension of the work on contact close binary systems stability would be a consideration of massive systems. Although theoretical investigations of massive contact binaries indicate that these systems should tend to have a mass ratio $q \simeq 1$, the observational evidence shows the opposite $q < 1$ (see Abdul-Masih et al. 2022, and references therein). The stability analysis of massive binaries similar to the one for low-mass systems, however, bears some potential problems. One particular question is whether the standard Roche model is a valid description of such systems (Schuerman 1972, Djurasevic 1986, Drechsel et al. 1995)? But even if we adopt this model, the primary may not be close to MS, as we generally assume for the low-mass systems, and the secondary will surely also be oversized but can no longer be treated as a low-mass fully convective star.

Nevertheless, a number of objects, such as magnetic massive stars (Schneider et al. 2019), Be stars (Shao and Li 2014), luminous blue variables or similar stars such as η Car (Smith et al. 2018), and peculiar Type-II supernovae like SN1987A (Menon and Heger 2017) have all been suggested to result in massive binary mergers. An interesting example is V838 Mon. It appeared as an atypical nova in 2002, and was later characterized as a luminous red nova (LRN). The erupting star was a cool extremely luminous supergiant (in the post-eruption phase designated as L-type Evans et al. 2003), with $L \gtrsim 10^6 L_\odot$ and radius reaching $R \gtrsim 10^3 R_\odot$ (Tylenda 2005). The nova that made V838 Mon temporarily the brightest star in the Milky Way, produced an iconic light echo (Bond et al. 2003) that helped to constrain the distance (Tylenda 2004) and thus the absolute parameters, but the question of the progenitor remained open. It is possible that nova V838 Mon was a merger in a triple system – merging close binary consisting of a B-type star + lower mass companion, in a binary orbit with another (survived) B-type star (Kamiński et al. 2021, and references therein). Similar events may be M31-RV (Rich et al. 1989) and M31LRN 2015 (Williams et al. 2015, Lipunov et al. 2017) in the Andromeda Galaxy (M31), and a small number of other extragalactic transients designated as LRN detected so far (Pastorello et al. 2019, Howitt et al. 2020).

4. CONCLUSION

The investigation of low mass ratio contact binaries of W UMa–type is a fruitful field of research, especially today when there is a growing interest in the astronomical community on binary mergers. As we have shown, under right conditions, a W UMa–type binary can reach critical separation, which can be related to the critical mass ratio below which we expect the components to merge (Arbutina 2007, 2009a,b). It is likely that some blue stragglers, the FK Com-type stars, and (luminous) red novae are produced in this way.

The critical mass ratio depends on the primary's (and secondary's) gyro-radius (Fig. 6). Assuming the primary a ZAMS star, we can link its gyro-radius to the mass through the $k_1^2 = k_1^2(M_1)$ relation. This relation shows that there is a minimum of k_1 at about $1.5M_\odot$ (Wadhwa et al. 2021), translating into minimum mass ratio

$$q_{\min} = 0.042 - 0.044,$$

depending on the filling factor ($f = 0-1$), for roughly that same mass $\sim 1.5M_\odot$ (Fig. (5)).

One should keep in mind that this is a global minimum and $q_{\rm inst}$ which decides whether the system is stable or unstable, is different for each particular binary (Wadhwa et al. 2023a) i.e. for each primary, in this simplified analysis. The gyro-radius–stellar mass relation should also include a metallicity dependence (Wadhwa et al. 2024). However, this relation is for ZAMS, and even if we neglect the question of the secondary, we could ask ourselves whether the primary is always close to ZAMS? The gyration radius can be even lower for the evolved MS stars (Jiang et al. 2010). For example, for the Sun $k_\odot^2 \approx 0.06$ (Allen 1973), while it is larger for an 1 $M_\odot$ ZAMS star ($k_1^2 \approx 0.087$). Thus it is not impossible that there are stable systems with mass ratio $q < q_{\min}$, but, since most of the primaries in the W UMa-type stars seem to be close to MS, $q_{\min}$ should be a reasonably good estimate (as well as the values for $q_{\rm inst}$).

On the observational side, there is a number of past or ongoing searches for merging systems (Kurtenkov 2017, Wadhwa et al. 2021, Li et al. 2021, Gazeas et al. 2021b, Li et al. 2022, Wadhwa et al. 2022a,b,c, Christopoulou et al. 2022, Popov and Petrov 2022, Liu et al. 2023, Wadhwa et al. 2023a,b). We can only hope that, in the near future, one of these searches will result into identification of an unstable system, such as V1309 Sco (Tylenda et al. 2011), that will allow us to have another nature's live broadcast of a stellar merger event.

Acknowledgements – BA acknowledges the funding provided by the Science Fund of the Republic of Serbia through project #7337 "Modeling Binary Systems That End in Stellar Mergers and Give Rise to Gravitational Waves" (MOBY), and by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia through the contract # 451-03-66/2024-03/200104.

REFERENCES

Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2016, PhRvL, 116, 061102

Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017, PhRvL, 119, 161101

Abdul-Masih, M., Sana, H., Hawcroft, C., et al. 2021, A&A, 651, A96

Abdul-Masih, M., Escorza, A., Menon, A., Mahy, L. and Marchant, P. 2022, A&A, 666, A18

Akerlof, C., Amrose, S., Balsano, R., et al. 2000, AJ, 119, 1901

Allen, C. W. 1973, Astrophysical quantities (London: Athlone Press)

Alton, K. B. 2018, JAAVSO, 46, 3

Alton, K. B. and Stepień, K. 2016, AcA, 66, 357

Alton, K. B. and Stepień, K. 2021, AcA, 71, 123

Arbutina, B. 2007, MNRAS, 377, 1635

Arbutina, B. 2009a, MNRAS, 394, 501

Arbutina, B. 2009b, PASP, 121, 1036
Ayres, T. R., Kashyap, V., Saar, S., et al. 2016, ApJS, 223, 5
Bernasconi, L. and Behrend, R. 2002, IBVS, 5234, 1
Binnendijk, L. 1965, Veroeffentlichungen der Remeis-Sternwarte zu Bamberg, 27, 36
Binnendijk, L. 1970, VA, 12, 217
Binnendijk, L. 1977, VA, 21, 359
Bond, H. E., Henden, A., Levay, Z. G., et al. 2003, Natur, 422, 405
Bond, H. E., Mink, J., Doane, A. and Lavallee, S. 2022, AJ, 164, 28
Bopp, B. W. and Rucinski, S. M. 1981, in Fundamental Problems in the Theory of Stellar Evolution, ed. D. Sugimoto, D. Q. Lamb, and D. N. Schramm, Vol. 93, 177
Bopp, B. W. and Stencel, R. E. 1981, ApJL, 247, L131
Caton, D., Gentry, D. R., Samec, R. G., et al. 2019, PASP, 131, 054203
Chen, X., Wang, S., Deng, L., de Grijs, R. and Yang, M. 2018, ApJS, 237, 28
Chen, X., Wang, S., Deng, L., et al. 2020, ApJS, 249, 18
Christopoulou, P.-E., Lalounta, E., Papageorgiou, A., et al. 2022, MNRAS, 512, 1244
Chugainov, P. F. 1966, IBVS, 172, 1
Cook, E. and Kobulnicky, H. 2024, in American Astronomical Society Meeting Abstracts, Vol. 56, American Astronomical Society Meeting Abstracts, 423.02
Cook, E., Kobulnicky, H., Molnar, L. and Henderson, L. 2022, in American Astronomical Society Meeting Abstracts, Vol. 54, American Astronomical Society Meeting #240, 205.05
Csizmadia, S. and Klagyivik, P. 2004, A&A, 426, 1001
Darwin, G. H. 1879, Obs, 3, 79
de Miguel, E. 2010, the International Variable Star Index, https://www.aavso.org/vsx/
Demartino, R., Kocyla, D., Predom, C. and Wetherbee, E. 1996, IBVS, 4322, 1
Djurasevic, G. 1986, Ap&SS, 124, 5
Drechsel, H., Haas, S., Lorenz, R. and Gayler, S. 1995, A&A, 294, 723
Eaton, J. A. 2016, MNRAS, 457, 836
Eggen, O. J. and Iben, Icko, J. 1989, AJ, 97, 431
Eggleton, P. 2006, Evolutionary Processes in Binary and Multiple Stars (Cambridge: Cambridge University Press)
Eggleton, P. P. 1971, MNRAS, 151, 351
Eggleton, P. P. 1972, MNRAS, 156, 361
Eggleton, P. P. 1983, ApJ, 268, 368
Eggleton, P. P. and Kiseleva-Eggleton, L. 2002, ApJ, 575, 461
Eggleton, P. P., Faulkner, J. and Flannery, B. P. 1973, A&A, 23, 325
Eker, Z. 1999, NewA, 4, 365
Evans, A., Geballe, T. R., Rushton, M. T., et al. 2003, MNRAS, 343, 1054
Ferreira, T., Saito, R. K., Minniti, D., et al. 2019, MNRAS, 486, 1220
Flannery, B. P. 1976, ApJ, 205, 217
Gazeas, K., Zola, S., Liakos, A., et al. 2021a, MNRAS, 501, 2897
Gazeas, K. D., Niarchos, P. G. and Zola, S. 2007, in Astronomical Society of the Pacific Conference Series, Vol. 370, Solar and Stellar Physics Through Eclipses, ed. O. Demircan, S. O. Selam, and B. Albayrak, 279
Gazeas, K. D., Loukaidou, G. A., Niarchos, P. G., et al. 2021b, MNRAS, 502, 2879
Gettel, S. J., Geske, M. T. and McKay, T. A. 2006, AJ, 131, 621
Guo, D., Li, K., Liu, F., et al. 2023a, PASP, 135, 044201
Guo, D. F., Li, K., Hu, S. M., et al. 2015, JApA, 36, 399
Guo, D.-F., Li, K., Gao, X., et al. 2020, MNRAS, 497, 3381
Guo, D.-F., Li, K., Liu, F., et al. 2022, MNRAS, 517, 1928
Guo, D.-F., Li, K., Liu, F., Li, H.-Z. and Liu, X.-Y. 2023b, MNRAS, 521, 51
Hilditch, R. W. 2001, An Introduction to Close Binary Stars (Cambridge: Cambridge University Press)
Hong, K., Lee, J. W., Kim, D.-J., et al. 2024, AJ, 167, 18
Howitt, G., Stevenson, S., Vigna-Gómez, A., et al. 2020, MNRAS, 492, 3229
Hubová, D. and Pejcha, O. 2019, MNRAS, 489, 891
Hut, P. 1980, A&A, 92, 167
Jiang, D., Han, Z., Wang, J., Jiang, T. and Li, L. 2010, MNRAS, 405, 2485
Kaluzny, J. and Shara, M. M. 1987, ApJ, 314, 585
Kamiński, T., Tylenda, R., Kiljan, A., et al. 2021, A&A, 655, A32
Kandulapati, S., Devarapalli, S. P. and Pasagada, V. R. 2015, MNRAS, 446, 510
Kjurkchieva, D. P. and Marchev, D. V. 2005, A&A, 434, 221
Kjurkchieva, D. P., Popov, V. A. and Petrov, N. I. 2018, RAA, 18, 129
Kjurkchieva, D. P., Popov, V. A. and Petrov, N. I. 2020, NewA, 77, 101352
Kuiper, G. P. 1941, ApJ, 93, 133
Kurtenkov, A. 2017, BlgAJ, 26, 26
Lalounta, E., Christopoulou, P. E., Papageorgiou, A., E., F. C. and Catelan, M. 2024, arXiv:2405.10614
Landin, N. R., Mendes, L. T. S. and Vaz, L. P. R. 2009, A&A, 494, 209
Latković, O., Čeki, A. and Lazarević, S. 2021, ApJS, 254, 10
Leiner, E., Mathieu, R. D., Vanderburg, A., Gosnell, N. M. and Smith, J. C. 2019, ApJ, 881, 47
Leiner, E. M. and Geller, A. 2021, ApJ, 908, 229
Li, K., Xia, Q.-Q., Kim, C.-H., et al. 2021, ApJ, 922, 122
Li, K., Gao, X., Liu, X.-Y., et al. 2022, AJ, 164, 202
Li, L. and Zhang, F. 2006, MNRAS, 369, 2001
Li, L., Han, Z. and Zhang, F. 2005, MNRAS, 360, 272
Lipunov, V. M., Blinnikov, S., Gorbovskoy, E., et al. 2017, MNRAS, 470, 2339
Liu, L., Qian, S.-B., Soonthornthum, B., et al. 2015, PASJ, 67, 74
Liu, X.-Y., Li, K., Michel, R., et al. 2023, MNRAS, 519, 5760

Loukaidou, G. A., Gazeas, K. D., Palafouta, S., et al. 2022, MNRAS, 514, 5528
Lucy, L. B. 1976, ApJ, 205, 208
Lucy, L. B. and Wilson, R. E. 1979, ApJ, 231, 502
Maceroni, C. and van't Veer, F. 1993, A&A, 277, 515
Martini, P., Wagner, R. M., Tomaney, A., et al. 1999, AJ, 118, 1034
Mason, E., Diaz, M., Williams, R. E., Preston, G. and Bensby, T. 2010, A&A, 516, A108
McCarthy, J. K. and Ramsey, L. W. 1984, ApJ, 283, 200
Menon, A. and Heger, A. 2017, MNRAS, 469, 4649
Merrill, P. W. 1948, PASP, 60, 382
Mochnacki, S. W. 1984, ApJS, 55, 551
Montesinos, B., Gimenez, A. and Fernandez-Figueroa, M. J. 1988, MNRAS, 232, 361
Muller, G. and Kempf, P. 1903, ApJ, 17, 201
Nakano, S., Nishiyama, K., Kabashima, F., et al. 2008, IAUC, 8972, 1
Nandez, J. L. A., Ivanova, N. and Lombardi, J. C., J. 2014, ApJ, 786, 39
O'Connell, D. J. K. 1951, Publications of the Riverview College Observatory, 2, 85
Paczynski, B. 1964, AJ, 69, 124
Panov, K. and Dimitrov, D. 2007, A&A, 467, 229
Papageorgiou, A., Christopoulou, P.-E., Lalounta, E., et al. 2023, ApJ, 952, 141
Pastorello, A., Mason, E., Taubenberger, S., et al. 2019, A&A, 630, A75
Pecaut, M. J. and Mamajek, E. E. 2013, ApJS, 208, 9
Pecaut, M. J., Mamajek, E. E. and Bubar, E. J. 2012, ApJ, 746, 154
Pepper, J., Stanek, K. Z., Pogge, R. W., et al. 2008, AJ, 135, 907
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565
Pešta, M. and Pejcha, O. 2023, A&A, 672, A176
Pintr, P. 2015, the International Variable Star Index, https://www.aavso.org/vsx/
Popov, V. A. and Petrov, N. I. 2022, NewA, 97, 101862
Poro, A., Paki, E., Blackford, M. G., et al. 2022, PASP, 134, 064201
Postnov, K. A. and Yungelson, L. R. 2014, Living Reviews in Relativity, 17, 3
Pribulla, T. and Rucinski, S. M. 2008, MNRAS, 386, 377
Pribulla, T., Rucinski, S. M., Blake, R. M., et al. 2009, AJ, 137, 3655
Qian, S., Yang, Y., Zhu, L., He, J. and Yuan, J. 2006, Ap&SS, 304, 25
Qian, S. B., Zhu, L. Y., Soonthornthum, B., et al. 2005, AJ, 130, 1206
Quadri, U., Strabla, L. and Girelli, R. 2017, JAAVSO, 45, 15
Rasio, F. A. 1995, ApJL, 444, L41
Rich, R. M., Mould, J., Picard, A., Frogel, J. A. and Davies, R. 1989, ApJL, 341, L51
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009
Rucinski, S. M. 1981, A&A, 104, 260
Rucinski, S. M. 1985a, in Interacting Binary Stars, ed. J. E. Pringle and R. A. Wade (Cambridge: Cambridge University Press), 85
Rucinski, S. M. 1985b, in Interacting Binary Stars, ed. J. E. Pringle and R. A. Wade (Cambridge: Cambridge University Press), 113
Rucinski, S. M. 1991, AJ, 101, 2199
Rucinski, S. M. 1992a, AJ, 103, 960
Rucinski, S. M. 1992b, AJ, 104, 1968
Rucinski, S. M. 1993, in Astrophysics and Space Science Library, Vol. 177, The Realm of Interacting Binary Stars, ed. J. Sahade, G. E. McCluskey, and Y. Kondo (Dordrecht: Springer), 111
Rucinski, S. M. 1998, AJ, 115, 1135
Rucinski, S. M. 2015, AJ, 149, 49
Schneider, F. R. N., Ohlmann, S. T., Podsiadlowski, P., et al. 2019, Natur, 574, 211
Schuerman, D. W. 1972, Ap&SS, 19, 351
Şenavcı, H. V., Doğruel, M. B., Nelson, R. H., Yılmaz, M. and Selam, S. O. 2016, PASA, 33, e043
Shao, Y. and Li, X.-D. 2014, ApJ, 796, 37
Skarka, M., Mašek, M., Brát, L., et al. 2017, Open European Journal on Variable Stars, 185, 1
Smith, N., Andrews, J. E., Rest, A., et al. 2018, MNRAS, 480, 1466
Srdoc, G. 2010, the International Variable Star Index, https://www.aavso.org/vsx/
Sriram, K., Malu, S., Choi, C. S. and Vivekananda Rao, P. 2016, AJ, 151, 69
Stepien, K. 2006a, AcA, 56, 199
Stepien, K. 2006b, AcA, 56, 347
Stępień, K. 2011, A&A, 531, A18
Strohmeier, W. 1967, IBVS, 195, 1
Tylenda, R. 2004, A&A, 414, 223
Tylenda, R. 2005, A&A, 436, 1009
Tylenda, R. and Kamiński, T. 2016, A&A, 592, A134
Tylenda, R., Hajduk, M., Kamiński, T., et al. 2011, A&A, 528, A114
Udalski, A. 2003, AcA, 53, 291
Wadhwa, S. S. 2005, Ap&SS, 300, 329
Wadhwa, S. S. 2006, Ap&SS, 301, 195
Wadhwa, S. S., De Horta, A., Filipović, M. D., et al. 2021, MNRAS, 501, 229
Wadhwa, S. S., De Horta, A., Filipović, M. D., et al. 2022a, RAA, 22, 105009
Wadhwa, S. S., De Horta, A. Y., Filipović, M. D., et al. 2022b, JApA, 43, 94
Wadhwa, S. S., de Horta, A. Y., Filipović, M. D. and Totohill, F. H. N. 2022c, JApA, 43, 42
Wadhwa, S. S., Arbutina, B., Petrović, J., et al. 2023a, PASP, 135, 094201
Wadhwa, S. S., Arbutina, B., Tothill, N. F. H., et al. 2023b, PASP, 135, 074202
Wadhwa, S. S., Landin, N. R., Kostić, P., et al. 2024, MNRAS, 527, 1
Walter, F. M. and Basri, G. S. 1982, ApJ, 260, 735
Webbink, R. F. 1976, ApJ, 209, 829
Williams, S. C., Darnley, M. J., Bode, M. F. and Steele, I. A. 2015, ApJL, 805, L18
Wilson, R. E. 2008, ApJ, 672, 575

Woźniak, P. R., Vestrand, W. T., Akerlof, C. W., et al. 2004, AJ, 127, 2436
Yakut, K. and Eggleton, P. P. 2005, ApJ, 629, 1055
Yang, Y.-G. and Qian, S.-B. 2015, AJ, 150, 69
Yin, Z.-X., Meng, Z.-B., Wu, P.-R., et al. 2023, RAA, 23, 085013
Zhang, X., Chen, X., Fu, J. and Li, Y. 2023, MNRAS, 523, 1394
Zhang, X.-D. 2024, NatSR, submitted
Zhang, X.-D. and Qian, S.-B. 2020, MNRAS, 497, 3493
Zhu, L.-Y., Zhao, E.-G. and Zhou, X. 2016, RAA, 16, 68
Zola, S., Rucinski, S. M., Baran, A., et al. 2004, AcA, 54, 299

КРИТИЧНИ ОДНОС МАСА ЗА КОНТАКТНЕ ДВОЈНЕ СИСТЕМЕ ТИПА W UMA

Бојан Арбутина[1] and **Surjit Wadhwa**[2]

[1]*Катедра за астрономију, Математички факултет, Универзитет у Београду Студентски трг 16, 11000 Београд, Србија*

E–mail: *arbo@astro.edu.rs*

[2]*School of Science, Western Sydney University, Locked Bag 1797, Penrith, NSW 2751, Australia*

УДК 52-335.7 + 524.387
Прегледни рад по позиву

Контактни системи представљају тесне двојне системе у којима обе компоненте испуњавају своје унутрашње Рошове овале, тако да су звезде у директном контакту и потенцијално размењују масу и енергију. Најчешћи такви системи мале масе су тзв. звезде типа W UMa. У последњих неколико година, постоји у астрономској заједници растуће интересовање за сударе звезда, пре свега због детекције гравитационих таласа (судари црних рупа и неутронских звезда), али и због алтернативног модела за супернове типа Ia (судар два бела патуљка) које су посебно значајне у космологији где су играле важну улогу у открићу тамне енергије и убрзаног ширења васионе. У том контексту, контактни системи типа W UMa са малим односом маса су посебно интересантни, будући да и у њиховом случају постоје индикације да долази до судара звезда и могућег формирања брзоротирајућих звезда типа FK Com, залуталих плавих звезда и (сјајних) црвених нових попут V1309 Sco. Наиме, ранија теоријска истраживања су показала да када је орбитални момент импулса свега око три пута већи од ротационог момента импулса ротације, долази до плимске Дарвинове нестабилности, услед које компоненте више не могу остати синхронизоване, орбита се брзо смањује и коначно долази до судара компонената и њиховог стапања у једну звезду. Поменути услов стабилности може се повезати са неким критичним односом маса испод којега очекујемо да систем буде нестабилан. У овом раду дајемо преглед тих услова и показујемо како се могу искористити за идентификацију система-кандидата за судар. На крају, разматрамо један број система са екстремно малим односом маса познатих у литератури, као и перспективу за будућа истраживања на ову тему.