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CORRELATION FUNCTION AND ELECTRONIC SPECTRAL LINE BROADENING IN RELATIVISTIC PLASMAS

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SUMMARY: The electrons dynamics and the time autocorrelation function $C_{EE}(t)$ for the total electric microfield of the electrons on positive charge impurity embedded in a plasma are considered when the relativistic dynamic of the electrons is taken into account. We have, at first, built the effective potential governing the electrons dynamics. This potential obeys a nonlinear integral equation that we have solved numerically. Regarding the electron broadening of the line in plasma, we have found that when the plasma parameters change, the amplitude of the collision operator changes in the same way as the time integral of $C_{EE}(t)$. The electron-impurity interaction is taken first as screened Deutsh interaction and second as Kelbg interaction. Comparisons of all interesting quantities are made with respect to the previous interactions as well as between classical and relativistic dynamics of electrons.

Key words. **plasmas – relativistic processes – line: profiles**

1. INTRODUCTION

The nonlinear behavior of electric charges around an impurity charge of the same sign is a problem that has been studied for a long time due to its great importance in many techniques (Holtsmark 1919, Hooper 1966, Iglesias et al. 1983, Boerker et al. 1987, Berkovsky et al. 1996). It is worth to mention that for fully ionized plasma composed of electrons and positive ions, the hypothesis of one component plasma (OCP) allows us to ignore the effects of ions motion with respect to those of electrons because the mass ratio is about $m_\mathrm{e}/m_\mathrm{i} \approx 1/2000$. So, the system is composed of only one kind of mobile charges (electrons), whereas the species of the opposite charge (ions) are modeled by the continuous background which provides electrical neutrality. Coulomb forces between point charges are purely repulsive and charges approach very close to each other only rarely whatever the plasma conditions. Concerning the ion-electron interaction, it is clear that it requires a quantum mechanical description. In this case, the Coulomb potential is replaced by a finite and regularized potential at the origin (Deutsh 1977, Deutsh et al. 1978, Minoo et al. 1981) $V_\mathrm{ie}^\mathrm{SD}(r)(\mathrm{Screened-Deutsh}) = -(Ze^2/r)(1-\exp(-r/\lambda_T))\exp(-r/\lambda_\mathrm{D})$ or Kelbg interaction (Filinov et al. 2003) $V_\mathrm{ie}^\mathrm{K}(r) = -\frac{Ze^2}{r\sqrt{\pi}}(1-\exp(-r^2/\lambda_T^2) + \sqrt{\pi}\frac{r}{\lambda_T}(1-\mathrm{erf}(\frac{r}{\lambda_T})))$ where $\lambda_\mathrm{D} = (k_\mathrm{B}T/(4\pi n_\mathrm{e}e^2))^{1/2}$, $\lambda_T = (2\pi\hbar^2/(m_\mathrm{e}k_\mathrm{B}T))^{1/2}$, and n_e is the density of the electrons and, in this way, the quantum effects are approximately taken into account. Furthermore, we will also consider these two potentials for the interaction between the electron and the continuous positive background. We note that many recent works on statistical properties of electrons in plasmas exist. For example, we find in Dufour et al. (2005) and Dufty et al. (2003) that the electrons are considered as classical particles moving by the first Newton law ($d\overrightarrow{v}/dt = -m_{\rm e}^{-1}\overrightarrow{\nabla}.V(r)$ where $m_{\rm e}$ is the electron rest mass). In addition, the interaction between the electron and the continuous background is taken as purely Coulombic. In our work, we consider the relativistic motion of the electron around the impurity to compute $C_{EE}(t)$. This task passes through two steps: - The first step: computation of the effective potential V(r) in which the electron moves, - The second step: we solve the relativistic equation of motion for the electron in the effective potential ($d\overrightarrow{p}/dt = -\overrightarrow{\nabla}.V(r)$ where $\overrightarrow{p} = m\overrightarrow{v}$ and $m = m_{\rm e}/(1-v^2/c^2)^{1/2}$). Furthermore, when we compute the effective potential V(r), we consider that the electron interacts with the test charge and with the continuous background positive charge via a regularized potential, whereas it interacts with the electrons via the Debye potential. In Section 2 we construct the equation for the effective potential which yields the subsequent results of this paper. We also solve this equation and present some discussions on its solutions. The dynamics properties, that is to say, the time autocorrelation function of the electron microfield, are presented in Section 3. Section 4 applies the results of Section 3 to the electronic broadening of the line shape in plasmas. At the end, we close this paper by Conclusion. Before starting the second Section, let us recall the relevant parameters for our study: the charge number Z, the average distance between electrons $a = (3/4\pi n_{\rm e})^{1/3}$, the electron coupling constant $\Gamma = e^2/(kTa)$, the electron de Broglie thermal length λ_T, the degree of quanticity $\eta = \lambda_T/a$ and the Debye length $\lambda_{\rm D}$. The cases considered in this work are (for Z=2, Z=4 and Z=8): the coupling parameter $\Gamma = 0.1$, the quanticity parameter $\eta = 0.177$, the dimensionless Debye length $\eta' = \lambda_{\rm D}/a = 1.826$. These parameters correspond to the electron density $n_{\rm e} = 2. \times 10^{20}{\rm cm}^{-3}$ and to the temperature $T = 1.6 \times 10^5$K. In this region of temperature and electron density, the non-relativistic treatment of the plasma becomes invalid (Mihajlov et al. 2011). We shall then treat the electron motion around the impurity in the framework of the relativistic classical mechanics.

2. INTEGRAL EQUATION FOR THE EFFECTIVE ENERGY POTENTIAL

2.1. Construction of integral equation for the effective energy potential

Let us consider a medium consisting of electrons and a continuous background of neutralizing positive electrical charges. This is the so called model of the one component plasma (OCP). At first, the distribution of the electrons is that of Maxwell-Boltzmann governing the equilibrium state of the electrons' system. If we place a positive ion of charge Ze (called test charge or impurity) at the coordinates origin the system is disturbed and, after a certain time t, it will reach a new equilibrium state described by a new distribution of electrons over the space around the charge Ze. The latter is determined through the potential energy of an electron located at distance r from the test charge Ze when the system reaches this new equilibrium state. This potential energy is built as a sum of three contributions:

$$V(r) = V_{\rm ie}(r) + V_{\rm ee}(r) + V_{\rm ef}(r) \tag{1}$$

where $V_{\rm ie}(r)$ is the potential energy of ion-electron interaction (the ion is the test charge), $V_{\rm ee}(r)$ is the interaction energy of the electron with all other electrons and $V_{\rm ef}(r)$ is the interaction energy of the electron with the continuous neutralizing background of ions (Kalman et al. 2002, Talin et al. 2002). The ion-electron interaction is taken in a way that we can consider the quantum effects at short distances; we represent it here by the following pseudo-potential (Deutsh 1977, Deutsh et al. 1978, Minoo et al. 1981):

$$V_{\rm ie}^{\rm SD}(r) = -\frac{Ze^2}{r}(1 - e^{-r/\lambda_T})e^{-r/\lambda_{\rm D}} \tag{2}$$

or Kelbg interaction:

$$V_{\rm ie}^{\rm K}(r) = -\frac{Ze^2}{r\sqrt{\pi}}.(1 - e^{-\frac{r^2}{\lambda_T^2}}) + \frac{r\sqrt{\pi}}{\lambda_T}(1 - {\rm erf}(\frac{r}{\lambda_T}))) \tag{3}$$

We will first investigate the case of screened Deutsh potential, while the results for the Kelbg potential are then straightforward. Most previous studies (Talin et al. 2002, Dufty et al. 2003, Dufour et al. 2005) took the Coulomb interaction like electrons interactions with themselves and with uniform neutralizing background of positive electric charge. To approach to the reality and taking into account the effect of screening in our study, we will take the electron-electron interaction to be that of the Debye potential energy $e^2e^{-r/\lambda_{\rm D}}/r$ such as the potential energy $V_{\rm ee}(r)$ in the mean field so that approximation is equal to:

$$V_{\rm ee}(r) = e^2 \int f(\overrightarrow{r'}, \overrightarrow{p}) \, \frac{e^{-\frac{|\overrightarrow{r} - \overrightarrow{r'}|}{\lambda_{\rm D}}}}{|\overrightarrow{r} - \overrightarrow{r'}|} \, d\overrightarrow{p}^3 d\overrightarrow{r'}^3, \tag{4}$$

where:

$$f(\overrightarrow{r}, \overrightarrow{p}) = \frac{N}{\Omega}(\frac{m\beta}{2\pi})^{3/2} e^{-\beta(\frac{\overrightarrow{p}^2}{2m} + V(r))} \tag{5}$$

is the Maxwell-Boltzmann distribution, N is the total number of electrons, and Ω is the volume of the system, whereas the energy potential of the electron with the positive background neutralizing charge is given by:

$$V_{\mathrm{ef}}^{\mathrm{SD}}(r) = -n_{\mathrm{e}}e^2 \int \frac{(1-e^{-\frac{|\vec{r}-\vec{r}'|}{\lambda_T}})e^{-\frac{|\vec{r}-\vec{r}'|}{\lambda_{\mathrm{D}}}}}{|\vec{r}-\vec{r}'|} d\vec{r}'^3 \tag{6}$$

In this formula, we have introduced the screened Deutsh interaction between the electron and the continuous background of positive charge. Then, the potential interaction energy of the electron, with all the plasma components, satisfies the following nonlinear integral equation:

$$\begin{aligned} V^{\mathrm{SD}}(r) &= V_{\mathrm{ie}}^{\mathrm{SD}}(r) + \\ & e^2 \int\int f(\vec{r}', \vec{p}) \frac{e^{-\frac{|\vec{r}-\vec{r}'|}{\lambda_D}}}{|\vec{r}-\vec{r}'|} d\vec{p}^3 d\vec{r}'^3 \\ & -n_{\mathrm{e}}e^2 \int \frac{(1-e^{-\frac{|\vec{r}-\vec{r}'|}{\lambda_T}})e^{-\frac{|\vec{r}-\vec{r}'|}{\lambda_{\mathrm{D}}}}}{|\vec{r}-\vec{r}'|} d\vec{r}'^3 \end{aligned} \tag{7}$$

Using spherical coordinates and some basic calculations, the last integral equation is transformed into the following:

$$\begin{aligned} V^{\mathrm{SD}}(r) &= V_{\mathrm{ie}}^{\mathrm{SD}}(r) + 2\pi n_{\mathrm{e}}e^2 \int_0^\infty \frac{r'dr'}{r}\lambda_{\mathrm{D}}. \\ & (e^{-(r+r')/\lambda_{\mathrm{D}}} - e^{-|r-r'|/\lambda_{\mathrm{D}}})(1-e^{-\beta V^{\mathrm{SD}}(r')}) \\ & -2\pi n_{\mathrm{e}}e^2 \int_0^\infty \frac{r'dr'}{r}\lambda(e^{-\frac{r+r'}{\lambda}} - e^{-\frac{|r-r'|}{\lambda}}) \end{aligned} \tag{8}$$

where $\lambda = \lambda_{\mathrm{D}}\lambda_T/(\lambda_T + \lambda_{\mathrm{D}})$.

In order to deal with an dimensionless equation, we put $Y^{\mathrm{SD}}(x) = -aV^{\mathrm{SD}}(r)/(Ze^2)$, $a = (4\pi n_{\mathrm{e}}/3)^{-1/3}$, $x = r/a$, $\eta = \lambda_T/a$, $\eta' = \lambda_{\mathrm{D}}/a$ and $\xi = \eta\eta'/(\eta+\eta')$. After that, we obtain the desired dimensionless integral equation:

$$\begin{aligned} Y^{\mathrm{SD}}(x) &= \tfrac{1}{x}(1-e^{-x/\eta})e^{-x/\eta'} \\ & -\tfrac{3}{2Z}\int_0^\infty \tfrac{t}{x}[\eta'(e^{-\frac{x+t}{\eta'}} - e^{-\frac{|x-t|}{\eta'}})(1-e^{Z\Gamma Y^{\mathrm{SD}}(t)}) \\ & -\xi(e^{-\frac{x+t}{\xi}} - e^{-\frac{|x-t|}{\xi}})]dt \end{aligned} \tag{9}$$

The same calculations for the case of Kelbg interaction give:

$$\begin{aligned} Y^{\mathrm{K}}(x) &= Y_{\mathrm{ie}}^{\mathrm{K}}(x) + \\ & \tfrac{3}{2Z}\int_0^\infty \tfrac{t}{x}[\eta'(e^{-\frac{x+t}{\eta'}} - e^{-\frac{|x-t|}{\eta'}})e^{Z\Gamma Y^{\mathrm{K}}(t)} \\ & -\tfrac{1}{\eta\sqrt{\pi}}(F(t+x) - F(|x-t|))]dt. \end{aligned} \tag{10}$$

where:

$$Y_{\mathrm{ie}}^{\mathrm{K}}(x) = \frac{1}{x\sqrt{\pi}}[1-e^{-(\frac{x}{\eta})^2} + \frac{x\sqrt{\pi}}{\eta}(1-\mathrm{erf}(\frac{x}{\eta}))] \tag{11}$$

and

$$\begin{aligned} F(x) &= -x(\eta + x\tfrac{\sqrt{\pi}}{2} + \tfrac{\eta}{2}e^{-(\frac{x}{\eta})^2}) + \\ & \eta^2\tfrac{\sqrt{\pi}}{2}erf(\tfrac{x}{\eta})(\tfrac{3}{2} + (\tfrac{x}{\eta})^2) \end{aligned} \tag{12}$$

It should be noted here that Shukla et al. (2008) has also studied the electron dynamics around an impurity by considering the hot and degenerate electrons. For this the quantum distribution of Thomas-Fermi was used.

2.2. Numerical solution of the integral equation for the potential energy

We can solve the nonlinear integral equation (9) by the method of successive iterations (fixed point method FPM) starting with the initial function $Y_0(x) = Y_{\mathrm{ie}}^{\mathrm{SD}}(x) = \frac{1}{x}(1-e^{-x/\eta})e^{-x/\eta'}$. We can also solve this integral equation by transforming it into a second order nonlinear differential equation and then use the method of Runge-Kutta (RKM) to solve it. The numerical solution of the nonlinear integral equations (9-10), in the case $\Gamma = 0.1$, $\eta = 0.177$, $\eta' = 1.826$ and $Z = 2$ and $Z = 8$ by the iterative method gives the potential energy as shown in Figs. 1-2.

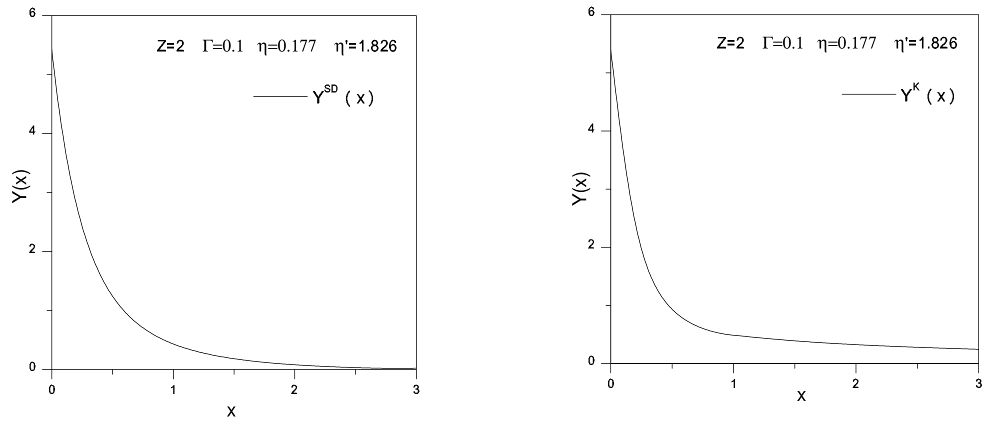

Fig. 1. *Effective potential energy of the electron for Z=2 in Deutsh and Kelbg cases.*

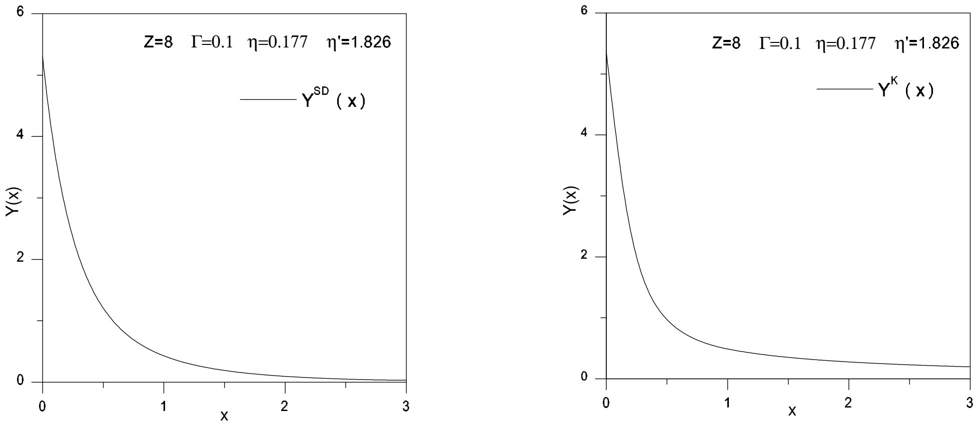

Fig. 2. *Effective potential energy of the electron for Z=8 in Deutsh and Kelbg cases.*

In Figs. 1-2, we notice that using the Kelbg potential as initial potential and interaction potential between the electron and the continuous background of positive charge in the integral equation, gives a solution decreasing more weakly than when screened Deutsh potential is used. This means that Deutsh solution is strongly screened, that is to say that its range is also inferior to that of Kelbg solution. The RKM allows us to solve, equivalently to the integral equation, the nonlinear differential equation for the Deutsh case. In this way we have solved the equation:

$$Y'' + \frac{2}{r}Y' = \frac{3}{Z}(e^{Z\Gamma Y(r)} - 1 + (\frac{\xi}{\eta'})^2) + \frac{1}{\eta'^2}Y(r) - \frac{1}{r\eta}(\frac{2}{\eta'} + \frac{1}{\eta})e^{-r(\frac{1}{\eta'}+\frac{1}{\eta})} \quad (13)$$

with initial conditions:

$$Y(0) \approx 1/\eta \ \ and \ \ Y'(0) \approx -(\eta'/2\eta + 1)/\eta\eta' \quad (14)$$

Fig. 3 shows this equivalence, for the Deutsh case, with the effective potential $Y^{\mathrm{SD}}(r)$ when Z=8, $\Gamma = 0.1$, $\eta = 0.177$, $\eta' = 1.826$.

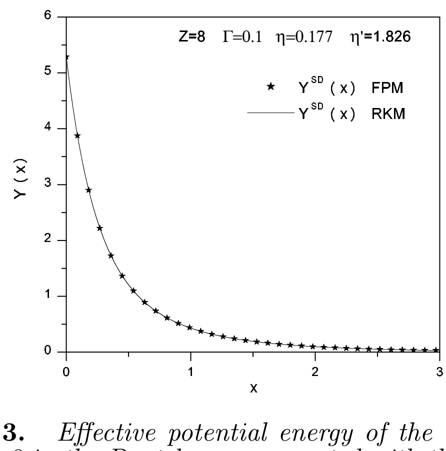

Fig. 3. *Effective potential energy of the electron for Z=8 in the Deutsh case computed with the Fixed Point Method (* * *) and Runge-Kutta Method (—).*

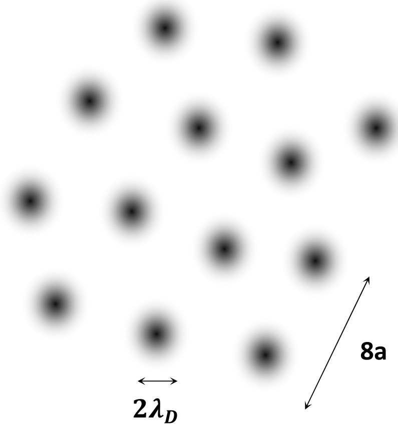

Fig. 4. *Schematic map of electron distribution around the impurities in the plasma.*

We have also found that the RKM (differential equation) is faster than the fixed point method (integral equation). The drawback of the RKM is that it is very sensitive to the initial conditions. Conversely, the FPM, despite that it requires a lot of time for computation, it has more guarantees that the result converges towards an exact solution. Another drawback of the FPM is that it is adaptable only for the shielded initial potential.

Mathematically, the integral Eq. (9) admits finite solutions at short distance (when r has approximately the screening length as shown in Fig. 4). This rapid convergence towards the solution is guaranteed by the screening effect. Physically, this may be interpreted by the following reasoning: a non bounded electron interacts with a neighborhood of some mean inter-electron distance a. This neighborhood contains electrons that are distributed with a density $n_{\mathrm{e}}(r)$ around a single impurity of positive electric charge. This means that the impurities are distributed in plasma with a mean constant density such that the neighborhood of each electron contains only a single impurity (see schematic map in Fig. 4).

What has just been said suggests that the numerical integration of the integral equation or the equation differential must be truncated to the size of this neighborhood. When the screening is weak (that is to say that λ_D is very large) the convergence towards the solution of the integral Eq. (9) becomes slow and the solution coincides at large distance with Kelbg solution, see Figs. 1-2.

3. THE DYNAMICAL PROPERTIES OF THE ELECTRONS

3.1. Relativistic electron trajectories in plasma

The calculation of real trajectories of relativistic electrons in a hot plasma is a necessary step to calculate several dynamical properties such as the time autocorrelation function, the diffusion coefficient and the electric permittivity... So, the purpose of this section is the calculation of relativistic trajectories of an electron in a plasma, and then we compare between classical and relativistic trajectories for few cases of potential energy. The relativistic force acting on an electron in the plasma is equal to the derivative of the momentum $\overrightarrow{P}$:

$$\overrightarrow{F} = \frac{d\overrightarrow{P}}{dt} = \frac{dm}{dt}\overrightarrow{v} + m\frac{d\overrightarrow{v}}{dt} \tag{15}$$

and the derivative of the mass is:

$$\begin{aligned} \tfrac{dm}{dt} &= m_e \tfrac{d}{dt}\left(1/\sqrt{1-v^2/c^2}\right) \\ &= m_e \tfrac{1}{c^2}\overrightarrow{v}\cdot\overrightarrow{\gamma}\left(1-v^2/c^2\right)^{-3/2} \end{aligned} \tag{16}$$

where $\overrightarrow{\gamma}$ is the relativistic acceleration, m_e is the rest mass of the electron and c the speed of light in vacuum. Therefore, this force is equal to:

$$\overrightarrow{F} = \frac{m_e}{c^2}\left(1-v^2/c^2\right)^{-3/2}\left(\overrightarrow{v}\cdot\overrightarrow{\gamma}\right)\overrightarrow{v} + \frac{m_e}{\sqrt{1-v^2/c^2}}\overrightarrow{\gamma} \tag{17}$$

and the force on the other hand equals:

$$\overrightarrow{F} = \frac{d\overrightarrow{P}}{dt} = -\overrightarrow{\nabla}V(r) \tag{18}$$

where $V(r)$ represents the potential energy (8) of an electron in the plasma at position r from the coordinate origin.

We write the Eqs. 17) and (18) in cartesian coordinates, and equating member to member we find the following system of equations:

$$\left\{\begin{aligned} \tfrac{m_e\omega^3}{c^2}v_x\left((v_x+\tfrac{c^2}{\omega^2 v_x})\gamma_x + v_y\gamma_y + v_z\gamma_z\right) &= -\tfrac{x}{r}\tfrac{\partial V(r)}{\partial r} \\ \tfrac{m_e\omega^3}{c^2}v_y\left(v_x\gamma_x + (v_y+\tfrac{c^2}{\omega^2 v_y})\gamma_y + v_z\gamma_z\right) &= -\tfrac{y}{r}\tfrac{\partial V(r)}{\partial r} \\ \tfrac{m_e\omega^3}{c^2}v_z\left(v_x\gamma_x + v_y\gamma_y + (v_z+\tfrac{c^2}{\omega^2 v_z})\gamma_z\right) &= -\tfrac{z}{r}\tfrac{\partial V(r)}{\partial r} \end{aligned}\right. \tag{19}$$

where: $\omega = 1/\sqrt{1-\left(v_x^2+v_y^2+v_z^2\right)/c^2} = 1/\sqrt{1-v^2/c^2}$

Solving this system of equations gives the expression for the acceleration as follows:

$$\overrightarrow{\gamma}: \left\{\begin{aligned} \gamma_x &= \zeta\left[+\left(c^2-v_x^2\right)x - yv_xv_y - zv_xv_z\right] \\ \gamma_y &= \zeta\left[-xv_xv_y + \left(c^2-v_y^2\right)y - zv_yv_z\right] \\ \gamma_z &= \zeta\left[-xv_xv_z - yv_yv_z + \left(c^2-v_z^2\right)z\right] \end{aligned}\right. \tag{20}$$

where: $\zeta = -\frac{1}{m_e c^2\omega}\frac{1}{r}\frac{\partial V(r)}{\partial r}$ and, now, we can use the following Taylor formula to find the trajectories of relativistic electrons:

$$\overrightarrow{r}(t+\Delta t) = 2\overrightarrow{r}(t) - \overrightarrow{r}(t-\Delta t) + (\Delta t)^2\,\overrightarrow{\gamma}(t) \tag{21}$$

In Figs. 5-6, we present the difference between classical and relativistic trajectories of an electron in plasma governed by the effective potential (9) and (10). These figures show that the motion of electrons in a cold plasma around the impurity center can be bound, but the electrons in the hot plasma are free due to their high velocities and the bounded trajectories do not appear in relativistic motion.

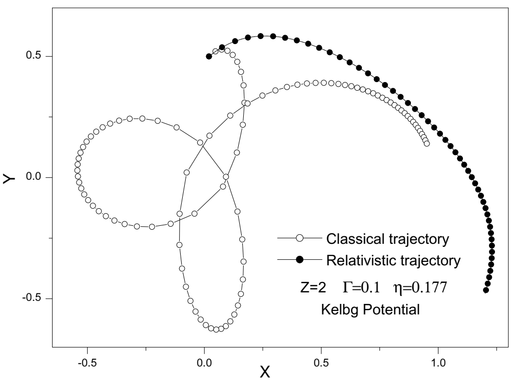

Fig. 5. *Classical and relativistic trajectories for the initial conditions: r(0)=0.7 and v(0)=0.485.*

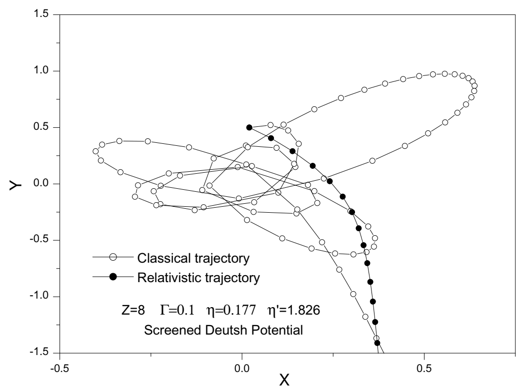

Fig. 6. *Classical and relativistic trajectories for the initial conditions: r(0)=0.76 and v(0)=1.04.*

3.2. The microfield autocorrelation function.

The total electric microfield due to the electrons on the impurity centered at the coordinate origin is given by:

$$\vec{E} = \sum_{k=1}^{N} \vec{e_{ie}}(r_k) \quad (22)$$

The dimensionless electric field auto-correlation function is given by Talin et al. (2008):

$$C_{EE}(t) = \frac{a^4}{e^2} \left\langle \vec{E}(t) \cdot \vec{E} \right\rangle$$

$$= \frac{a^4}{e^2} \int d\vec{r}_1 d\vec{v}_1 ....d\vec{r}_N d\vec{v}_N \vec{E} \cdot \vec{E}(-t)\rho_e$$

$$= \frac{a^4}{e^2} \int d\vec{r}_1 d\vec{v}_1 \vec{e}(r_1) N$$

$$\int d\vec{r}_2 d\vec{v}_2 ....d\vec{r}_N d\vec{v}_N \vec{E}(-t)\rho_e$$

$$= \frac{a^4}{e^2} \int d\vec{r}_1 d\vec{v}_1 \vec{e}(r_1) \Psi(\vec{r}_1, \vec{v}_1, t) \quad (23)$$

where ρ_e is the equilibrium canonical ensemble and $e(r_\alpha)$ is the single particle field. The integrations over degrees of freedom $2...N$ in the second equality define a reduced function $\Psi(r_1, v_1, t)$, which is the first member of a set of such functions

$$\Psi(\vec{r}_1, \vec{v}_1 ....\vec{r}_s, \vec{v}_s, t) =$$

$$N^s \int d\vec{r}_{s+1} d\vec{v}_{s+1} ....d\vec{r}_N d\vec{v}_N \vec{E}(-t)\rho_e. \quad (24)$$

It is straightforward to verify that these functions satisfy the BBGKY hierarchy (Van Kampen et al. 1967):

$$(\partial_t + \vec{v} \cdot \vec{\nabla}_r - \frac{(1-\frac{v^2}{c^2})^{1/2}}{m_e} \{\vec{\nabla}_r [V_{ie}(r) + V_{ef}(r)]\}$$
$$\cdot \{\vec{\nabla}_v - \frac{\vec{v}}{c}(\frac{\vec{v}.\vec{\nabla}_v}{c})\}) \Psi(\vec{r}, \vec{v}, t)$$
$$= \frac{1}{m_e \omega} \int d\vec{r}_1 d\vec{v}_1 [\vec{\nabla}_r V_{ee}(\vec{r} - \vec{r}_1)]$$
$$\cdot \{\vec{\nabla}_v - \frac{\vec{v}}{c}(\frac{\vec{v}.\vec{\nabla}_v}{c})\} \Psi^{(2)}(\vec{r}, \vec{v}; \vec{r}_1, \vec{v}_1, t) \quad (25)$$

where m_e is the electron mass at the rest and c is the speed of light. Recognizing this linear relationship, the basic approximation for weak coupling among the electrons is to neglect all of their correlations at all times:

$$\Psi^{(2)}(\vec{r}_1, \vec{v}_1, \vec{r}_2, \vec{v}_2, t) \to f(\vec{r}_2, \vec{v}_2) \Psi(\vec{r}_1, \vec{v}_1, t)$$
$$+ f(\vec{r}_1, \vec{v}_1) \Psi(\vec{r}_2, \vec{v}_2, t) \quad (26)$$

Use of (26) in the first hierarchy equation (25) gives directly the kinetic equation:

$$(\partial_t + L)\, \Psi(\vec{r}, \vec{v}, t) =$$
$$\frac{1}{m_e \omega} [\vec{\nabla}_v - \frac{\vec{v}}{c^2}(\vec{v}.\vec{\nabla}_v)] f(\vec{r}, \vec{v}) \cdot$$
$$\vec{\nabla}_r \int d\vec{r}_2 V_{ee}(\vec{r} - \vec{r}_2) \int d\vec{v}_2 \Psi(\vec{r}_2, \vec{v}_2, t) \quad (27)$$

where:

$$L = \vec{v}.\vec{\nabla} - \frac{1}{m_\mathrm{e}\omega} \left[\vec{\nabla}_r V(r)\right] .(\vec{\nabla}_v - \frac{\vec{v}}{c^2}(\vec{v}.\vec{\nabla}_v) \quad (28)$$

We limit ourselves to the solution of the homogeneous Eq. (27) which is given by:

$$\Psi(\vec{r}, \vec{v}, t) = f(\vec{r}, \vec{v}) \vec{e}_{mf}(\vec{r}(t)) \quad (29)$$

where:

$$\vec{e}_{mf}(\vec{r}) = \frac{1}{Ze} \vec{\nabla} V(r) \quad (30)$$

and $f(r, v)$ is the Maxwell-Juttner-Boltzmann distribution given by:

$$f(r, v) = \frac{\exp(-(mc^2 + V(r))/kT)}{m_\mathrm{e}^3 c^3 K_2(m_\mathrm{e} c^2/kT)}. \quad (31)$$

Here $m = m_\mathrm{e}\omega$ and $K_2(x)$ is the modified Bessel function. Replacing (29) in (23) we find:

$$C_{EE}(t) = \frac{a^4}{e^2} \int f(\vec{r}, \vec{v})\, \vec{e}(r) \cdot \vec{e}_{mf}(r(t)) d\vec{r} d\vec{v} \quad (32)$$

where $\vec{r}(t)$ is the time-dependent position vector. To get it for any time t, we have solved numerically (using Verlet algorithm) the equation of motion $d\vec{P}/dt = -e.\vec{e}_{mf}(r)$ where $\vec{P} = m_\mathrm{e}\vec{v}/\sqrt{(1-v^2/c^2)}$ is the relativistic momentum of the electron. In the calculation of $C_{EE}(t)$ the average on the velocities is done over the relativistic distribution $f(r, v)$.

Regarding the function $C_{EE}(t)$ in Fig. 7, we found: - when one moves away from $t = 0$, the relativistic effect is manifested more clearly. - In Fig. 8, we note that when we increase the charge number Z, the covariance $C(0)$ also increases, but all the relativistic curves in Kelbg and Deutsh cases decrease more quickly and have the same behavior for large values of time (vanishe quickly and never cross the Ox axis).

4. APPLICATION TO THE ELECTRONIC BROADENING IN PLASMAS

First, we have to keep in mind that (Alexiou 1994) and predecessors GBKO (Griem et al. 1962) considered the interaction between the electrons and the emitter in the impact approximation. They begin by outlining the process that led Alexiou (1994), Griem et al. (1962) to construct with hyperbolic trajectories a valid electronic collision operator for isolated lines. They considered the interaction of a plasma electron with an ion as purely Coulombic, and in addition that the motion of the plasma electron is due solely to the Coulomb field created by the ion. The field of the electron in an ion emitter is then Coulombic. We now introduce some new notations. First, we use the time-dependent interaction between a single plasma electron and emitter electron as follows:

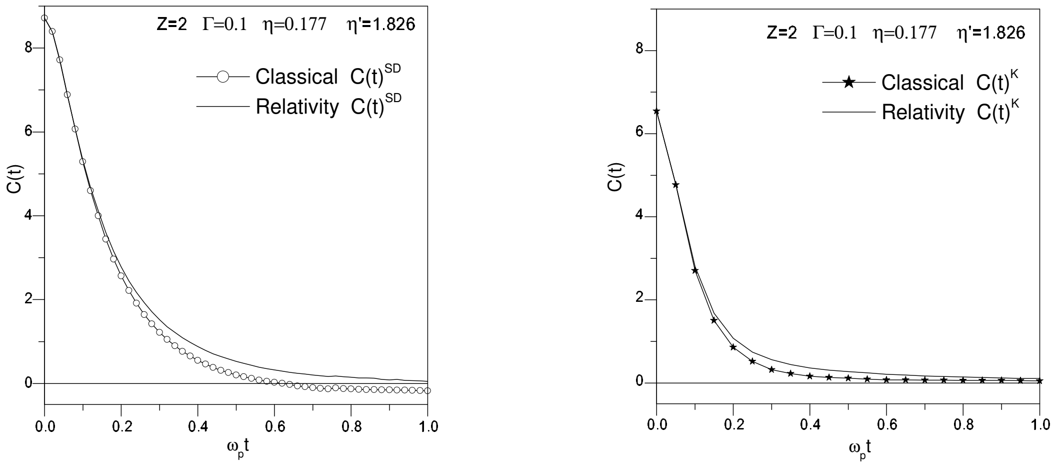

Fig. 7. *Classical and relativistic electric field auto-correlation function for Z=2 in Deutsh and Kelbg cases.*

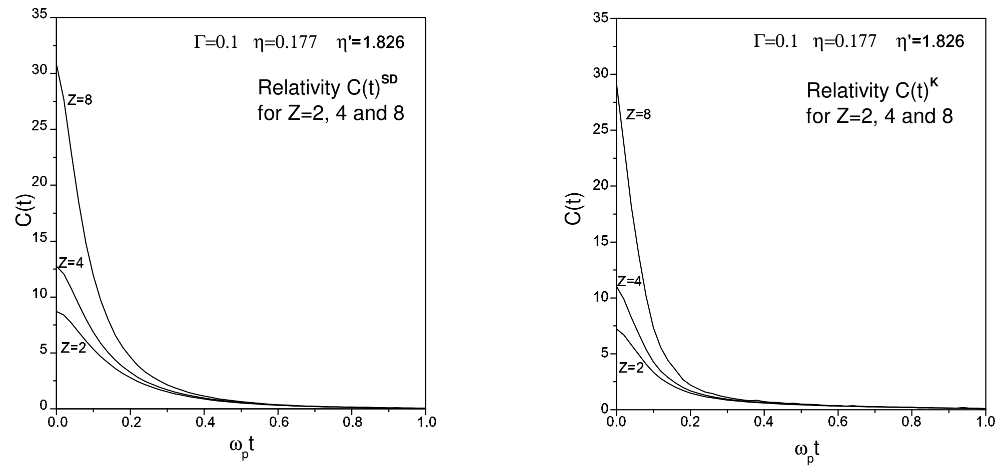

Fig. 8. *Relativistic electric field auto-correlation function for different Z in Deutsh and Kelbg cases.*

$$V(t) = \overrightarrow{d}.\overrightarrow{e}(t) = -e\overrightarrow{R}.\overrightarrow{e}(\overrightarrow{r}(t)) \qquad (33)$$

where $\overrightarrow{R}$ is the position vector of the emitter electron and $\overrightarrow{e}(t)$ is the individual electric field on the impurity due to the electron located at $\overrightarrow{r}$. Now we want to separate the purely atomic part and the part that depends on the details of the collision process. This leads to defining two quantities (Alexiou 1994):

$$\phi_d = -2\pi\frac{n_e e^2}{3\hbar^2}\int vF(v)dv\int \rho d\rho\int_{-\infty}^{\infty}dt_1 \int_{-\infty}^{t_1}dt_2 e^{i\omega_1 t_1}e^{i\omega_2 t_2}\overrightarrow{e}(t_1).\overrightarrow{e}(t_2) \qquad (34)$$

and:

$$\phi_{int} = -2\pi\frac{n_e e^2}{3\hbar^2}\int vF(v)dv\int \rho d\rho\int_{-\infty}^{\infty}dt_1 \int_{-\infty}^{\infty}dt_2 e^{i\omega_1 t_1}e^{i\omega_2 t_2}\overrightarrow{e}(t_1).\overrightarrow{e}(t_2) \qquad (35)$$

where $F(v)$ is the Maxwell distribution of the velocities. The integral:

$$2\pi n_e\int vF(v)dv\int \rho d\rho \qquad (36)$$

defines the average in the phases space of the particle positions $\vec{r}_i$ and the particle momentum $\vec{p}_i$. The distribution is that of Maxwell-Boltzmann $F(v)\exp(-V/kT)$, then we can transform ϕ_d as:

$$\phi_d = -\frac{e^2}{3\hbar^2}\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{t_1}dt_2 e^{i\omega_1 t_1+i\omega_2 t_2} < \overrightarrow{E}(t_1).\overrightarrow{E}(t_2) >_{\mathrm{can}} \qquad (37)$$

where $\overrightarrow{E}(t)$ is the total field created by the electrons of plasma on the emitter. By making use of the stationarity:

$$C_{EE}(t_1 - t_2) = < \overrightarrow{E}(t_1).\overrightarrow{E}(t_2) >_{\rm can} \quad (38)$$

we can check that:

$$\phi_d = -\frac{e^4}{3\hbar^2 a^4}\int_0^{\infty} C_{EE}(t)dt \quad (39)$$

(in s^{-1} unit) where $C_{EE}(t)$ is given by (32). The same formula is used by (Nguyen et al. 1967) for the collision operator:

$$\phi = -\overrightarrow{R_n}.\overrightarrow{R_n}(\frac{e^4}{3.\hbar^2 a^4})\int_0^{+\infty} C_{EE}(t)dt \equiv \overrightarrow{R_n}.\overrightarrow{R_n}\phi_d \quad (40)$$

In computing the collision operator ϕ, conversely to Alexiou (1994) and his predecessors, the plasma electron (the perturber) moves in the effective field created by the entire plasma. Moreover, this electron creates a field(Deutsh or Kelbg) at the impurity ion. Then we call ϕ_d the amplitude of the collision operator because it is this quantity that contains the plasma parameters through the correlation function $C_{EE}(t)$. This contains all the information regarding the density $n_{\rm e}$, the temperature T, and the charge number Z of ions. We present in Table 1 the ratio of the amplitudes of the collision operator between the classical and relativistic case for different plasma conditions such as $n_{\rm e}$, T and Z. We find that when Z is increased, the relativistic effect increases.

Table 1. The ratio $\phi_d^{\rm classical}/\phi_d^{\rm relativistic}$.

Z	Deutsh	Kelbg
1	0.9052	0.9154
2	0.8102	0.8263
4	0.5767	0.6554
8	0.3256	0.3396

From this table, we see that the difference between the classical and relativistic collision operator becomes more important when the number of charge Z increases.

5. CONCLUSION

The electrons dynamics and the time autocorrelation function $C_{EE}(t)$ for the total electric microfield of the electrons on positive charge impurity embedded in a plasma are considered when the relativistic dynamics of the electrons is taken into acount. We have, first, built the effective potential governing the electrons dynamics. This potential obeys a nonlinear integral equation which we solved numerically. We found, for fixed $\Gamma = 0.1$ and fixed density $n_{\rm e}$, that the relativistic effect becomes important (for $Z = 8$, $\phi_{\rm classical}/\phi_{\rm relativistic} \simeq 1/3$) when the charge number Z increases. The collision operator, responsible for electronic broadening in plasma, is investigated. The result is that when the plasmas parameters change, the amplitude of the collision operator becomes important. The electron-impurity interaction is firs taken to be the screened Deutsch interaction and then the Kelbg interaction. Comparisons of all relevant quantities are made for these interactions as well as between classical and relativistic dynamics of electrons. The relativistic trajectories of the plasma electrons around the impurity are more complicated the classical trajectories as it can be seen in Figs. 5-6. This fact has a direct effect on the behavior of the electric auto-correlation function. Indeed, when we move away from $t = 0$, the difference between the classical C_{EE} and the relativistic C_{EE} becomes more obvious.

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КОРЕЛАЦИОНА ФУНКЦИЈА И ЕЛЕКТРОНСКО ШИРЕЊЕ ЛИНИЈЕ У РЕЛАТИВИСТИЧКИМ ПЛАЗМАМА

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Оригинални научни рад

Разматрају се динамика електрона и временски зависна аутокорелациона функција $C_{EE}(t)$ за укупно електрично микро-поље електрона на позитивно наелектрисаној примеси урођеној у плазму, када се узима у обзир релативистичка динамика електрона. Најпре смо конструисали ефективни потенцијал који управља динамиком електрона. Овај потенцијал се покорава нелинеарној интегралној једначини коју смо решили нумерички. Што се тиче електронског ширења линије у плазми, нашли смо да када се параметри плазме мењају, амплитуда сударног оператора се мења на исти начин као и интеграл по времену од $C_{EE}(t)$. Интеракција електрон-примеса је најпре узета као екранирана Деусова (Deutsh) интеракција, а затим као Келбгова (Kelbg) интеракција. Направљено је поређење свих интересантних величина са претходним интеракцијама, као и класичне и релативистичке динамике електрона.