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TD-88Up – UPGRADED NEUTRAL EARTH'S THERMOSPHERE TOTAL DENSITY TD-88 MODEL

B. Šurlan[1] and S. Šegan[2]

[1]*Mathematical Institute of the Serbian Academy of Sciences and Arts,*
Kneza Mihaila 36, 11001 Belgrade, Serbia

E–mail: *brankica74@matf.bg.ac.yu*

[2]*Department of Astronomy, Faculty of Mathematics, University of Belgrade*
Studentski trg 16, 11001 Belgrade, Serbia

E–mail: *ssegan@matf.bg.ac.yu*

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SUMMARY: Improved constants of the total density TD-88 model for the Earth's neutral thermosphere are calculated. The model is fully functional within the height range of 200 to 500 km, with fixed values of the mean solar flux and geomagnetic index. The control data of the atmospheric air density are derived from the aeronomical NRLMSISE-00 model which was used as the reference one. The upgraded TD-88, named TD-88Up, model is obtained by the extended LSQ method with varying all model parameters.

Key words. Earth

1. INTRODUCTION

Since the times when the first artificial satellite was launched, it was realized that the changes of orbital elements of the Low Earth Orbit (LEO) satellite are connected with variations in the upper atmosphere. These changes are caused by the atmospheric drag and this perturbing force has become a very powerful tool for studying the upper layers of planetary atmospheres.

By numerical integration of the equations of the satellite perturbed motion or by analytical methods, the total density of the atmosphere around the satellite's path can be obtained. On the other hand, to predict orbits of LEO satellites with the satisfactory accuracy, it is necessary to know the distribution and variation of the atmospheric density which can be obtained from an assumed atmospheric model.

The most commonly used models are the aeronomical ones, which are very precise and effective for certain purposes, but in the theory of satellite motion many difficulties in their application appear. Because of this, a better way is to define an analytical or semi-analytical model of atmospheric density distribution. This enables an analytical treatment of the perturbations of the satellite orbital elements and, using the reverse procedure, to obtain the model parameters, too.

Such a model is the Total Density (TD) model of the Earth's neutral thermosphere, and some variants of it (Sehnal 1988, Sehnal and Pospíšilová 1988, Bezděk and Vokrouhlický 2004).

The advantages of such models with respect to the aeronomical ones, especially in the theory of satellite motion, are the reason for their continuous developing on the basis of very precise aeronomical models. In our previous paper (Šegan and Šurlan 2005) we showed how to improve the TD-88 model using the up-to-date aeronomical NRLMSISE-00 model (Picone et al. 2002). An improved agreement among these models was achieved by fitting their differences in the case with only two variable model parameters. However, an even better agreement is achieved by varying all the parameters simultaneously.

2. THE ATMOSPHERIC DENSITY MODELS

The TD-88 model (Sehnal and Pospíšilová 1988) is an improved TD model (Sehnal 1988). The air density (ρ) is described by the expression

$$\rho = f_x f_0 k_0 \sum_{n=1}^{7} h_n g_n \ , \qquad (1)$$

where

$$\begin{aligned} f_x &= 1 + a_1(F_x - F_b), \\ f_0 &= a_2 + f_m, \\ f_m &= (F_b - 60)/160, \\ k_0 &= 1 + a_3(K_p - 3). \end{aligned}$$

To include the solar and geomagnetic effects and, moreover, the individual terms containing factors dependent on the mean solar flux, the coefficients g_n are used. Through these coefficients the average density, the individual mean solar flux dependence, the north-south asymmetry, annual and semi-annual, diurnal and semi-diurnal variations are included. The altitude dependence is described by the h_n terms,

$$h_n = K_{n,0} + \sum_{j=1}^{3} K_{n,j} \exp\left(\frac{120 - h}{29j}\right) \ , \qquad (2)$$

and the functions g_n are given by

$$\begin{aligned} g_1 &= 1 \\ g_2 &= f_m/2 + a_4 \\ g_3 &= \sin(d - p_3)\sin\varphi \\ g_4 &= (a_5 f_m + 1)\sin(d - p_4) \\ g_5 &= (a_6 f_m + 1)\sin 2(d - p_5) \\ g_6 &= (a_7 f_m + 1)\sin(t - p_6)\cos\varphi \\ g_7 &= (a_8 f_m + 1)\sin 2(t - p_7)\cos^2\varphi. \end{aligned}$$

The symbols denote: $K_{n,j}$, a_i – numerical constants of a model, p_n – phases, F_x – solar flux measured at 10.7 cm for previous day, F_b – mean solar flux averaged over three solar rotations, K_p – geomagnetic index 3 hours before the current local time, h – altitude, φ – latitude, and t – local time.

On the other side, the NRLMSISE-00 model is an empirical model of the Earth's neutral atmosphere valid from the ground up to the height of 1000 km of altitude. The densities for a given day in a year (IYD), the universal time in seconds (SEC), the altitude (ALT), latitude (GLAT), longitude (GLONG), local time (STL), the mean solar flux averaged over three solar rotations (F107A), the solar flux measured at 10.7 cm for previous day (F107) and the geomagnetic index (Ap) can be found at the Naval Research Laboratory web site.[1]

3. THE UPGRADING PROCEDURE

Till now, several improvements of the Sehnal's basic TD model have been made (Sehnal and Pospíšilová 1988, Bezděk and Vokrouhlický 2004). In this work, we first determined the $K_{n,j}$ model constants. The parameters a_i and p_n are considered as known, and for them we use the same values as in the TD-88 model. The procedure consisted of the following steps:

(i) Linearizing the density expression (1) with respect to $K_{n,j}$.
(ii) Varying all the parameters simultaneously.
(iii) Determining the new $K_{n,j}$ constants of the model, and their standard deviations (within an altitude range from 200 to 500 km).
(iv) Estimating accuracy of the obtained model constants.
(v) Comparing with the NRLMSISE-00 model.

We choose the longitude value GLONG=0° as a reference one; the variation of density with longitude was not considered, because this type of variation was not included in the TD-88 model. The universal time is expressed in seconds (SEC=STL·3600). The solar flux for the previous day is calculated from

$$\mathrm{F107} = (1. + 0.15N)\mathrm{F107A} \ , \ N = -1, 0, 1. \qquad (3)$$

We have used the K_p index rather than the A_p index as the parameter of geomagnetic activity.

The boundary values of intervals of physical and geometrical parameters and their steps and number of values are presented at Table 1. This led us to explore 221760 different density values as input data in our attempt to upgrade TD-88 model.

By extending the ordinary least square method (LSQ) and applying the linearization of the density expression (1) we computed the coefficients and their deviations for the upgraded model from

$$\sigma^2_{n,j} = \frac{D_{ii}}{D}\sigma^2_0 \ , \ n = \overline{1,7}, \ j = \overline{1,4}, \ i = \overline{1,28}, \qquad (4)$$

where D is the determinant of the matrix of the system, D_{ii} are algebraic complements of the diagonal elements ii and σ^2_0 is the standard deviation

$$\sigma^2_0 = \frac{1}{N - p}\sum_{k=1}^{N} \varepsilon_k \ , \ N = 5460, \ p = 28, \qquad (5)$$

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[1] `http://uap-www.nrl.navy.mil/uap/?code=7643;content=nrlmsise00.dist17`

Table 1. The boundary values of intervals of physical and geometrical parameters, their steps and number of values for the first iteration.

Parameter	Boundary values	Step [unit]	Number of values
IYD	$\langle 1; 331\rangle$	33 [day]	11
SEC	STL·3600	[s]	1
ALT	$\langle 200; 500\rangle$	50 [km]	7
GLAT	$\langle -80; 80\rangle$	40 [°]	5
GLONG	0	[°]	1
STL	$\langle 1; 22\rangle$	3 [h]	8
F107A	$\langle 60; 260\rangle$	40 [10^{-22} Wm^{-2}Hz^{-1}]	6
F107	Eqn. (3)	[10^{-22} Wm^{-2}Hz^{-1}]	3
K_p	$\langle 0; 9\rangle$	3	4

Table 2. The boundary values of intervals of physical and geometrical parameters, their steps and number of values which give the most accurate results.

Parameter	Boundary values	Step [unit]	Number of values
IYD	$\langle 1; 364\rangle$	33 [day]	12
SEC	STL·3600	[s]	1
ALT	$\langle 200; 500\rangle$	50 [km]	7
GLAT	$\langle -90; 90\rangle$	45 [°]	5
GLONG	0	[°]	1
STL	$\langle 0; 24\rangle$	2 [h]	13
F107A	150	[10^{-22} Wm^{-2}Hz^{-1}]	1
F107	150	[10^{-22} Wm^{-2}Hz^{-1}]	1
K_p	3		1

where ε_k are the residuals obtained by changing the calculated values of $K_{n,j}$ in the conditional equation, N is the total number of data, and p is the number of unknown parameters. All calculations are made by FORTRAN90 routines.

4. RESULTS

In the first solution of the system of normal equations and densities calculation with the new set of constants, a small number of densities (about 1%) was negative. We checked the numerical correctness of the algorithms used and all matrix transformations. Statistically, the results with negative densities were with smaller scattering, but we had to find the cause for these nonrealistic values.

As a next step, we therefore analysed variations of the parameters, and we found that variations of the geomagnetic index and solar flux are those that give rise to nonrealistic densities. Further analysis of these variations and conditional equations of LSQ confirmed that for any combination of the geomagnetic index and solar flux we can determine set of $K_{n,j}$ constants which give realistic densities.

Using the procedure of upgrading described above and exact values of the average solar flux and geomagnetic index obtained from measurements (or model) we can determine the constants $K_{n,j}$. Our set of constants yields densities which agree better with the NRLMSISE-00 model than those of the TD-88 model.

By an additional analysis of the resulting densities, obtained by changing values of the geomagnetic index and solar flux, we concluded that the most accurate results are obtained for $F_x = F_b = 150$, $K_p = 3$ ($A_p = 15$) and varying the other four parameters (Table 2). In this section we present the set of constant which we determined for this case.

With these values of parameters we have derived density values of the upgraded TD-88 model. The constants $K_{n,j}$ and their error estimates are shown in Tables 3 and 4. In this way we succeeded to upgrade the TD-88 model within the height range from 200 to 500 km. The upgrade of model we named the TD-88Up model.

5. COMPARISON OF THE NRLMSISE-00, TD-88, AND TD-88Up MODELS

In order to estimate the accuracy of the TD-88Up model we compared this model with the NRLMSISE-00 and the TD-88 models. The relative (percentage) deviation (δ) for $N = 5460$ selected points with a combination of parameters given in Table 2 is used to obtain total δ for the model. The formula used here is

Table 3. Constants $K_{n,j}$ for the TD-88Up model.

n/j	0	1	2	3
1	$0.133266 \cdot 10^{-08}$	$0.167935 \cdot 10^{-07}$	$0.678445 \cdot 10^{-08}$	$-0.558459 \cdot 10^{-08}$
2	$-0.405992 \cdot 10^{-08}$	$-0.400823 \cdot 10^{-07}$	$-0.195238 \cdot 10^{-07}$	$0.171885 \cdot 10^{-07}$
3	$-0.199071 \cdot 10^{-13}$	$-0.439091 \cdot 10^{-09}$	$-0.374988 \cdot 10^{-10}$	$-0.116933 \cdot 10^{-10}$
4	$0.407227 \cdot 10^{-14}$	$-0.250279 \cdot 10^{-10}$	$-0.434513 \cdot 10^{-11}$	$-0.151686 \cdot 10^{-11}$
5	$0.147905 \cdot 10^{-13}$	$-0.595860 \cdot 10^{-10}$	$-0.666754 \cdot 10^{-11}$	$-0.275309 \cdot 10^{-11}$
6	$-0.578693 \cdot 10^{-14}$	$-0.164250 \cdot 10^{-09}$	$0.521976 \cdot 10^{-10}$	$0.383611 \cdot 10^{-10}$
7	$0.117458 \cdot 10^{-13}$	$-0.185037 \cdot 10^{-10}$	$0.665665 \cdot 10^{-11}$	$-0.207915 \cdot 10^{-12}$

Table 4. Standard deviation $\sigma^2_{n,j}$ of constants $K_{n,j}$ for the TD-88Up model.

n/j	0	1	2	3
1	$-0.47242 \cdot 10^{-18}$	$-0.14135 \cdot 10^{-15}$	$-0.17066 \cdot 10^{-15}$	$0.56410 \cdot 10^{-17}$
2	$-0.43818 \cdot 10^{-17}$	$-0.13113 \cdot 10^{-14}$	$-0.15828 \cdot 10^{-14}$	$0.52321 \cdot 10^{-16}$
3	$0.28125 \cdot 10^{-24}$	$0.84428 \cdot 10^{-20}$	$0.36202 \cdot 10^{-20}$	$0.63290 \cdot 10^{-21}$
4	$0.70778 \cdot 10^{-26}$	$0.21247 \cdot 10^{-21}$	$0.91104 \cdot 10^{-22}$	$0.15927 \cdot 10^{-22}$
5	$0.72667 \cdot 10^{-26}$	$0.21814 \cdot 10^{-21}$	$0.93534 \cdot 10^{-22}$	$0.16352 \cdot 10^{-22}$
6	$0.31438 \cdot 10^{-24}$	$0.94372 \cdot 10^{-20}$	$0.40466 \cdot 10^{-20}$	$0.70744 \cdot 10^{-21}$
7	$0.66498 \cdot 10^{-26}$	$0.19962 \cdot 10^{-21}$	$0.85595 \cdot 10^{-22}$	$0.14964 \cdot 10^{-22}$

$$\delta = \frac{1}{N}\sum_{k=1}^{N}\delta_k = \frac{1}{N}\sum_{k=1}^{N} 100\frac{|\rho_{1k} - \rho_{2k}|}{\rho_{2k}} \ , \qquad (6)$$

where ρ_{1k} are the densities obtained from either the TD-88Up model or the TD-88, ρ_{2k} are the densities obtained from the NRLMSISE-00 model, and N is the total number of data.

The standard deviation (σ) of the TD-88Up model is given by

$$\sigma = \frac{1}{N}\sum_{k=1}^{N}\sigma_k = \frac{1}{N}\sum_{k=1}^{N}\sqrt{(\rho_{1k} - \rho_{2k})^2} \ . \qquad (7)$$

The calculated values of δ and σ for the TD-88 model are given in Table 5.

As it can be seen, with the new $K_{n,j}$ constants we attained a significantly better accuracy and a better agreement with the international standards. By the analysing the sigma values of the the TD-88Up model we concluded that the model agrees better with the NRLMSISE-00 model for altitudes 200–400 km and the largest differences between models appear for altitudes 400–500 km.

Table 5. Relative and standard deviation of the TD-88Up and the TD-88 models with respect to the NRLMSISE-00 model.

Model	δ [%]	σ^2
TD-88Up	7.14	$0.00356 \cdot 10^{-26}$
TD-88	20.92	$0.00236 \cdot 10^{-24}$

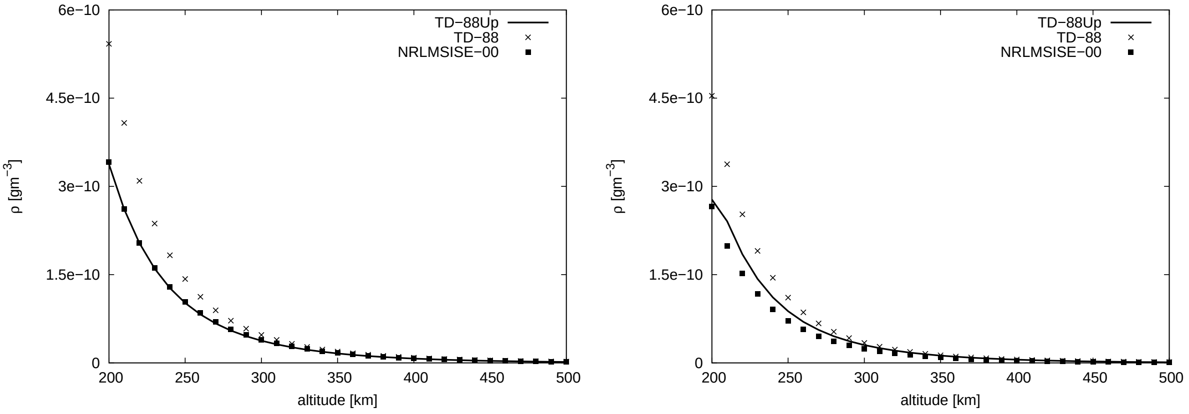

Fig. 1. *Total density dependence on the altitude for the models TD-88Up , TD-88, and NRLMSISE-00 for* $h \in \langle 200; 500\rangle$, $F_x = F_b = 150$, $K_p = 3$; *(Left)* $d = 80$, $t = 14$, $\varphi = 0$; *(Right)* $d = 330$, $t = 18$, $\varphi = 90$.

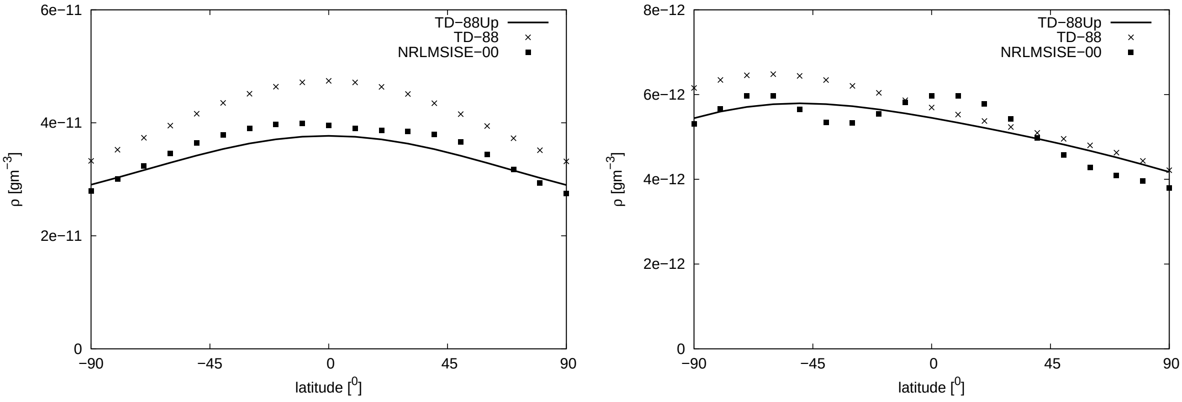

Fig. 2. *Latitudinal dependence of the total density for the models TD-88Up, TD-88, and NRLMSISE-00 for* $\varphi \in \langle -90; 90\rangle$*,* $F_x = F_b = 150$*;* $K_p = 3$*; (Left)* $d = 80$*,* $t = 14$*,* $h = 300$*; (Right)* $d = 330$*,* $t = 18$*,* $h = 400$*.*

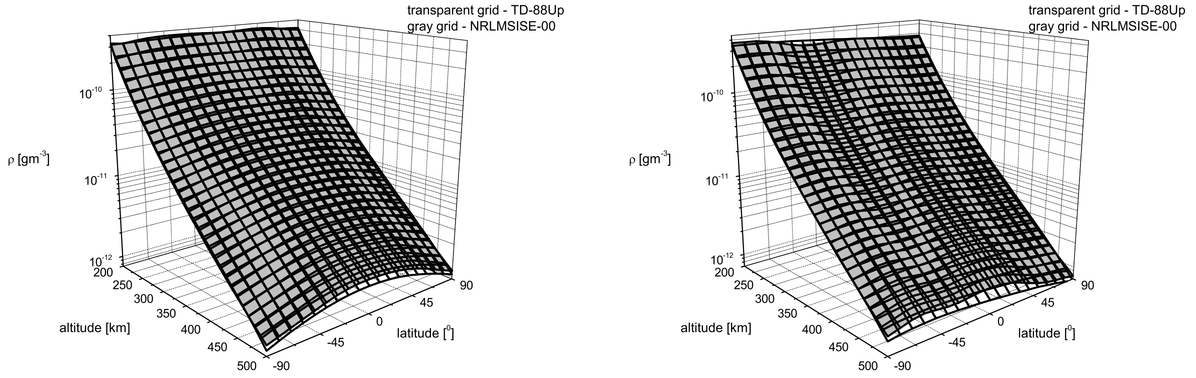

Fig. 3. *Total density dependence on the altitude and latitude for the TD-88Up (transparent grid) and NRLMSISE-00 (gray grid) models for* $\varphi \in \langle -90; 90\rangle$*,* $h \in \langle 200; 500\rangle$*,* $F_x = F_b = 150$*,* $K_p = 3$*; (Left)* $d = 80$*,* $t = 14$*; (Right)* $d = 330$*,* $t = 18$*.*

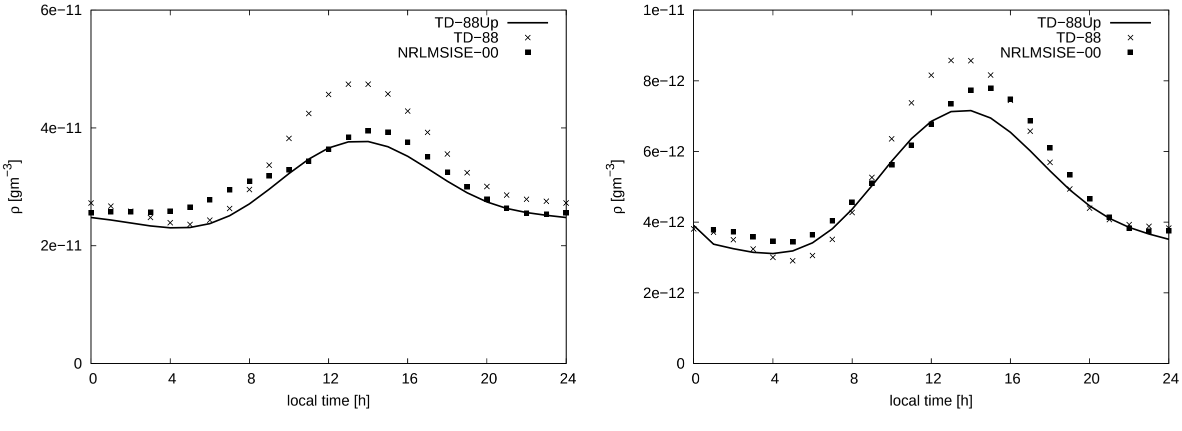

Fig. 4. *Local time dependence of the total density for TD-88Up, TD-88, and NRLMSISE-00 models for* $t \in \langle 0; 24\rangle$*,* $F_x = F_b = 150$*,* $K_p = 3$*; (Left)* $d = 80$*,* $\varphi = 0$*,* $h = 300$*; (Right)* $d = 330$*,* $\varphi = 90$*,* $h = 400$*.*

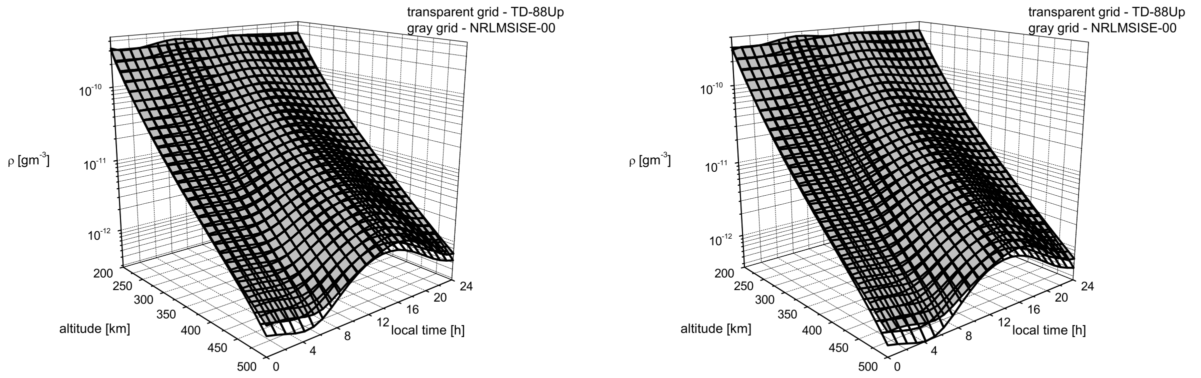

Fig. 5. *Total density dependence on the altitude and on the local time as evaluated on the basis of the TD-88Up (transparent grid) and NRLMSISE-00 (gray grid) models for* $t \in \langle 0; 24 \rangle$, $h \in \langle 200; 500 \rangle$, $F_x = F_b = 150$, $K_p = 3$; *(Left)* $d = 80$, $\varphi = 0$; *(Right)* $d = 330$, $\varphi = 90$.

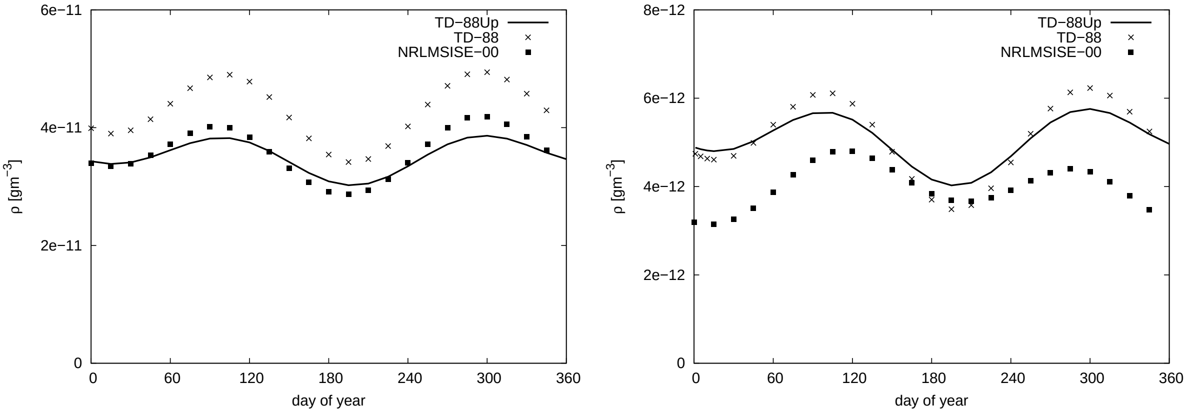

Fig. 6. *Seasonal (day of year) dependence of total density obtained from the TD-88Up, TD-88, and NRLMSISE-00 models for* $d \in \langle 0; 360 \rangle$, $F_x = F_b = 150$, $K_p = 3$; *(Left)* $t = 14$, $\varphi = 0$, $h = 300$; *(Right)* $t = 18$, $\varphi = 90$, $h = 400$.

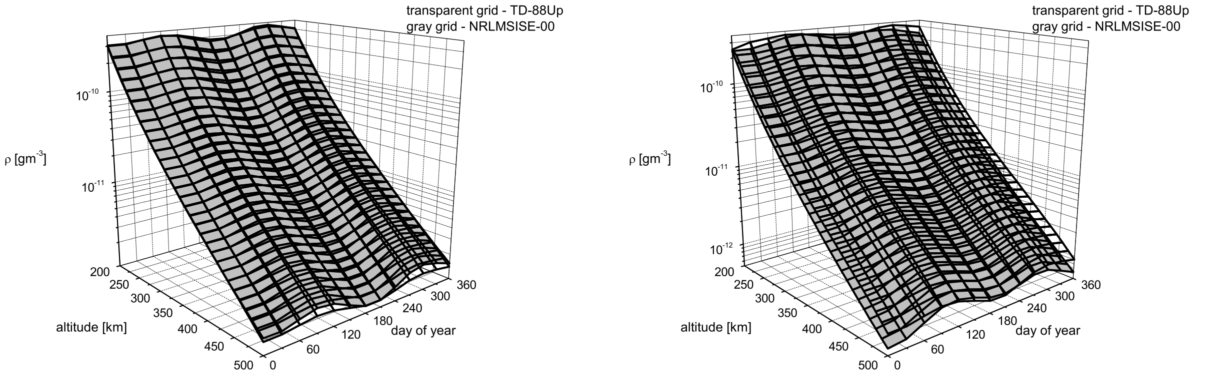

Fig. 7. *Depedence of the total density on the altitude and on the day of year obtained by the TD-88Up (transparent grid) and NRLMSISE-00 (gray grid) models for* $d \in \langle 0; 360 \rangle$, $h \in \langle 200; 500 \rangle$, $F_x = F_b = 150$, $K_p = 3$; *(Left)* $t = 14$, $\varphi = 0$; *(Right)* $t = 18$, $\varphi = 90$.

Since the NRLMSISE-00 model is very precise and includes many short-term density variations (see Figs. 2 and 4) which were not present in the previous empirical models, for example in the DTM model (Barlier et al. 1977) which was used for deriving constants of the TD-88 model, a good fitting of the TD-88 model to this model is a difficult task. Better agreement with the NRLMSISE-00 model could be obtained by correcting the analytical expression for the density (1), which is, however, beyond the scope of this paper.

Figs. 1 to 7 represent selected cases of models comparison. It can be seen from these figures that the agreement between models is not the same for any values of parameters.

Figs. 3, 6, and 7 display comparison of height density variations of TD-88Up and NRLMSISE-00 for varying latitude, local time, and day of the year. Regions where the models agree well and, consequently, where the plotted grids overlap, can be seen as "lighter" ones, whereas regions with some disagreement appear "darker".

In Fig. 2 one can see that densities obtained from the NRLMSISE-00 model show variations with latitude, which are not very well described by the TD- 88Up model. Also, in Fig. 4 densities obtained from the NRLMSISE-00 model show variations with local time, not only semi-diurnal but also terr-diurnal variations. For better agreement, these variations with latitude shoud be better analyticaly described in the TD-88Up model and terr-diurnal variations should be included.

6. CONCLUSIONS

The TD-88Up model gives better agreement with up-to-date data (the NRLMSISE-00 model) than the original TD-88 model. More comprehensive model can be achieved by increasing the number of terms in density expression, especially diurnal and terr-diurnal (diurnal-part) terms.

It is also possible to make some additional corrections of density variations by analyzing data regarding the atmospheric drag and in situ data from satellites.

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TD–88Up – УНАПРЕЂЕНИ МОДЕЛ ТОТАЛНЕ ГУСТИНЕ НЕУТРАЛНЕ ТЕРМОСФЕРЕ ЗЕМЉЕ

B. Šurlan[1] and S. Šegan[2]

[1]*Mathematical Institute of the Serbian Academy of Sciences and Arts,*
Kneza Mihaila 36, 11001 Belgrade, Serbia

E–mail: *brankica74@matf.bg.ac.yu*

[2]*Department of Astronomy, Faculty of Mathematics, University of Belgrade*
Studentski trg 16, 11001 Belgrade, Serbia

E–mail: *ssegan@matf.bg.ac.yu*

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Оригинални научни рад

Урађена је поправка коефицијената TD-88 модела неутралне термосфере Земље. Модел је потпуно функционалан у опсегу висина од 200 до 500 km, са фиксним вредностима средњег соларног флукса и геомагнетног индекса. Контролни подаци атмосферских густина изведени су из аерономског NRLMSISE-00 модела, који је коришћен као референтни. Проширеном методом најмањих квадрата са варирањем свих параметара модела, добијен је унапређен модел, означен са TD-88Up.