Solving Hierarchical Diffusion Equation Using Some Matrix Algebra

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Let $n_0,\ n_1,\ldots$ be a sequence of positive integers, $m_k := \prod_{j=0}^{k-1} n_j$, $m_0 := 1$ be orders of M-adic integers $\mathbb{Z}_M = \left\{ x = \sum_{j=0}^{\infty} x_j m_j\ ,\ x_j \in \{0,1,\ldots,n_j-1\} \right\}$ with non-archimedean norm $|x|_M = m_N^{-1}$, if $N = \min\{j \in \mathbb{N}_0\ :\ x_j \neq 0\}$. The ring $\mathbb{Z}_M$ is coincide with $\mathbb{Z}_p$ when $n_j = p$ for every $j \in \mathbb{N}_0$. Let $\rho\ :\ \mathbb{Q} \to \mathbb{Q}$ be a positive function such that $\rho(0) = 0$, $\rho(|i-j|_M) = \rho_{ij} \geq 0$ be a transition probability of a particle from state i to state j. Denote by Q the matrix of $(\rho_{ij})_{i,j=0}^{m_n-1}$. The matrix has a Parisi form

$$Q = \left(\begin{array}{cc|cc|cccc}
0 & \rho_0 & \rho_1 & \rho_1 & \rho_2 & \cdots & \rho_2 & \rho_2 \\
\rho_0 & 0 & \rho_1 & \rho_1 & \rho_2 & \cdots & \rho_2 & \rho_2 \\
\hline
\rho_1 & \rho_1 & 0 & \rho_0 & \rho_2 & \cdots & \rho_2 & \rho_2 \\
\rho_1 & \rho_1 & \rho_0 & 0 & \rho_2 & \cdots & \rho_2 & \rho_2 \\
\hline
\rho_2 & \rho_2 & \rho_2 & \rho_2 & 0 & \rho_0 & \rho_1 & \rho_1 \\
\vdots & \vdots & \vdots & \vdots & \rho_0 & 0 & \rho_1 & \rho_1 \\
\rho_2 & \rho_2 & \rho_2 & \rho_2 & \rho_1 & \rho_1 & 0 & \rho_0 \\
\rho_2 & \rho_2 & \rho_2 & \rho_2 & \rho_1 & \rho_1 & \rho_0 & 0
\end{array} \right)$$

see [1] or [2] for details. Let λ is an eigenvalue of Q corresponding to the constant function $P(t,x)$ as a function in x. The hierarchical diffusion equation

$$\frac{\partial P(t,j)}{\partial t} = (Q - \lambda I)P(t,j)\ ,\ j = 0,1,\ldots,m_n-1$$

is considered and explicit solution is obtained with help matrix algebra generated by $I_0,\ I_1,\ldots,I_n$. Here I_0 is identical matrix, and for any $k = 1,\ldots,n$ $I_k = \mathrm{diag}\{\underbrace{E_{m_k\times m_k},\ldots,E_{m_k\times m_k}}_{m_n/m_k}\}$ is a $m_n \times m_n$ block-diagonal matrix where $E_{m_k\times m_k}$ is a $m_k \times m_k$ matrix of 1. The matrix algebra method was at first used and described in [3], [4] for some interpolation problem.

References

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